ROM2F/2010/17

On stable higher spin states in Heterotic String Theories

Massimo Bianchi, **Luca Lopez**, **Robert Richter**
Dipartimento di Fisica, Università di Roma "Tor Vergata"
I.N.F.N. Sezione di Roma "Tor Vergata"
Via della Ricerca Scientifica, 00133 Roma, ITALY

arXiv:1010.1177v1 [hep-th] 6 Oct 2010

Abstract

We study properties of 1/2 BPS Higher Spin states in heterotic compactifications with extended supersymmetry. We also analyze non BPS Higher Spin states and give explicit expressions for physical vertex operators of the first two massive levels. We then study on-shell tri-linear couplings of these Higher Spin states and confirm that BPS states with arbitrary spin cannot decay into lower spin states in perturbation theory. Finally, we consider scattering of vector bosons off higher spin BPS states and extract form factors and polarization effects in various limits.

Introduction

One of the remarkable properties of String Theory – probably its hallmark – is the presence of an infinite tower of massive higher spin (HS) excitations in the free spectrum. Most of these are unstable and can decay into lower spin states after turning on interactions [1, 2, 3, 4]. Some are long-lived and can be detected at accessible energies if the string scale is much lower than the Planck scale [5, 6, 7, 8, 9, 10, 11]. In some – albeit unrealistic – configurations however, HS states can be perturbatively stable thanks to BPS conditions. In fact the very existence of these BPS states is a peculiarity of String Theory: no smooth supergravity solutions can describe supersymmetric states with spinning horizon in $D = 4$ [12, 13, 14]. Precisely for this reason their existence creates a puzzling situation insofar as the microscopic counting of the degeneracy of states is matched with the area of a stretched horizon, relying on Wald's generalization of Bekenstein-Hawking entropy formula [15, 16].

Aim of this note is to study properties of 1/2 BPS higher spin states within the perturbative spectrum of heterotic string compactifications on tori [17] and (freely acting) orbifolds [18]. With a bold abuse of language one could dub these states as 'small spinning black-holes' [19, 15, 16] in that they have a finite microscopic 'entropy', despite the lack of a semi-classical supergravity description [20, 21, 22, 23, 24]. As we will see, their spin and large degeneracy characterize their couplings to massless states, *e.g.* gravitons and vector bosons. In line with previous observations [1, 2, 3, 4], one should accept to abandon a semi-classical description and determine the nature of these peculiar string states in terms of observables such as scattering amplitudes, decay rates and form factors. In this note, relying on our previous work [25], we will take a further step in this direction. Contrary to recent analyses [5, 6, 7, 8, 9, 10, 11] of unstable string excitations in phenomenologically viable scenari with TeV scale tension and large extra dimension [26, 27], we will mostly focus on cases with at least $\mathcal{N} = 2$ whereby BPS condition guarantee the perturbative stability of higher spin states. One may hope that sectors with extended supersymmetry could be embedded in realistic chiral models.

The plan of the paper is as follows. In Section 1, we review the structure of $\mathcal{N} = 4$ 1/2 BPS multiplets with higher spin. We identify HS multiplets of this kind in various heterotic compactifications and discuss their vertex operators and BRST conditions. We extend the analysis to non BPS HS states in Section 2, where we display physical vertex operators for the first two massive levels. We then study on-shell tri-linear couplings of HS states in Section 3. Not unexpectedly, we find that BPS states with arbitrary spin cannot decay into lower spin states in perturbation theory. To study their properties one should consider at least 4-point scattering amplitudes: this is our task in Section 4. We also extract form factors and polarization effects at low energies. Finally we conclude with a summary of our results, open issues and puzzles in Section 5.

1 Higher Spin 1/2 BPS $\mathcal{N}=4$ multiplets in $D=4$

Heterotic strings [28, 29] compactified on six dimensional tori $\mathbf{T}^6$ enjoy $\mathcal{N}=4$ supersymmetry in $D=4$ [17, 30, 31, 32, 33, 34, 35]. The spectrum includes super-multiplets with arbitrarily high spin. Some of these multiplets saturate the BPS bound $M^2 = |\mathbf{p_L}|^2$, where p_L^i with $i=1,...6$ denote the central charges the gravi-photons couple to, and are shorter than generic long multiplets. In the perturbative spectrum only 1/2 BPS and long multiplets are present. Bosonic charged states in 1/2 BPS multiplets with maximal spin correspond to the vertex operators

$$V^{(-1)}_{\mathcal{H}_i} = \mathcal{H}_{i,\mu_1\cdots\mu_s} e^{-\varphi}\psi^i e^{i\mathbf{p}_L\mathbf{X}_L}\bar{\partial}X_R^{\mu_1}\cdots\bar{\partial}X_R^{\mu_s} e^{i\mathbf{p}_R\mathbf{X}_R}e^{ipX} \tag{1}$$

with internal excitations in the ground state of the L-moving sector, and

$$V^{(-1)}_{\mathcal{H}_\mu} = \mathcal{H}_{\mu,\mu_1\cdots\mu_s} e^{-\varphi}\psi^\mu e^{i\mathbf{p}_L\mathbf{X}_L}\bar{\partial}X_R^{\mu_1}\cdots\bar{\partial}X_R^{\mu_s} e^{i\mathbf{p}_R\mathbf{X}_R}e^{ipX} \tag{2}$$

with space-time excitations in the ground state of the L-moving sector. Clearly, at a given mass level there exist additional 1/2 BPS multiplets with lower spin.
By construction, the tensors $\mathcal{H}_{i,\mu_1\ldots\mu_s}$ and $\mathcal{H}_{\mu,\mu_1\ldots\mu_s}$ are totally symmetric in the non-compact space-time indices μ_i and, in order for the states to be BRST invariant, they should satisfy

$$p_L^i\mathcal{H}_{i,\mu_1\cdots\mu_s} = p^{\mu_1}\mathcal{H}_{i,\mu_1\mu_2\cdots\mu_s} = \eta^{\mu_1\mu_2}\mathcal{H}_{i,\mu_1\mu_2\mu_3\cdots\mu_s} = 0 \tag{3}$$

$$p^{\mu}\mathcal{H}_{\mu,\mu_1\cdots\mu_s} = p^{\mu_1}\mathcal{H}_{\mu,\mu_1\mu_2\cdots\mu_s} = \eta^{\mu_1\mu_2}\mathcal{H}_{\mu,\mu_1\mu_2\mu_3\cdots\mu_s} = 0\ . \tag{4}$$

The tensor $\mathcal{H}_{i,\mu_1\ldots\mu_s}$ accounts for five[1] states of spin s, while $\mathcal{H}_{\mu,\mu_1\ldots\mu_s}$ gives rise to spin $s+1$, s and $s-1$ states according to

$$\square\otimes\square\square\square\square\square\square = \square\square\square\square\square\square\square \oplus \begin{smallmatrix}\square\square\square\square\square\square\\ \square\end{smallmatrix} \oplus \square\square\square\square\square\ . \tag{5}$$

The reason why the second Young tableaux on the right-hand side of (5) corresponds to spin s is that, because of the BRST conditions (3), μ_i are effectively $SO(3)$ indices, thus $\begin{smallmatrix}\square\\ \square\end{smallmatrix} \equiv \square$. Therefore the number of bosonic degrees of freedom is

$$n_B = 2(s+1)+1+(5+1)(2s+1)+2(s-1)+1 = (2s+1)8_B\ . \tag{6}$$

Vertex operators for the fermionic states with maximal spin read

$$V^{(-1/2)}_{\Psi_{\alpha A}} = \Psi_{\alpha A,\mu_1\cdots\mu_s} e^{-\varphi/2}S^\alpha\Sigma^A e^{i\mathbf{p}_L\mathbf{X}_L}\bar{\partial}X_R^{\mu_1}\cdots\bar{\partial}X_R^{\mu_s} e^{i\mathbf{p}_R\mathbf{X}_R}e^{ipX} \tag{7}$$

[1] After imposing BRST conditions (3) the internal index i is allowed to run only over the five directions orthogonal to the central charge vector $\mathbf{p}_L$.

and

$$V^{(-1/2)}_{\overline{\Psi}_{\dot\alpha A}} = \overline{\Psi}^A_{\dot\alpha,\mu_1\cdots\mu_s} e^{-\varphi/2} C^{\dot\alpha} \Sigma^*_A e^{i\mathbf{p}_L\mathbf{X}_L} \bar\partial X^{\mu_1}_R \cdots \bar\partial X^{\mu_s}_R e^{i\mathbf{p}_R\mathbf{X}_R} e^{ipX} \ , \tag{8}$$

where S^α, $C^{\dot\alpha}$ are $SO(1,3)$ spin fields and Σ^A, Σ^*_A are $SO(6)$ spin fields. BRST invariance requires

$$p^\mu \bar\sigma^{\dot\alpha\alpha}_\mu \Psi_{\alpha A,\mu_1\cdots\mu_s} + p_{iL}\tau^i_{AB}\overline{\Psi}^{B\dot\alpha}_{\mu_1\cdots\mu_s} = 0 \tag{9}$$

that allows to express $\overline{\Psi}^{A\dot\alpha}_{\mu_1\cdots\mu_s}$ in terms of $\Psi_{\alpha A,\mu_1\cdots\mu_s}$

$$\overline{\Psi}^{A\dot\alpha}_{\mu_1\cdots\mu_s} = \frac{1}{M^2} p^\mu \bar\sigma^{\dot\alpha\alpha}_\mu p^i_L \bar\tau^{AB}_i \Psi_{\alpha B,\mu_1\cdots\mu_s} \tag{10}$$

with $M^2 = -p\cdot p = |\mathbf{p_L}|^{\mathbf{2}}$. Combing spin 1/2 from left-movers with spin s from Right-movers one gets $s+1/2$ and $s-1/2$. Taking into account the degeneracy $\mathbf{4}$ of $SO(5) \sim Sp(4) \subset SO(6)$, the number of fermionic degrees of freedom turns out to be

$$n_F = 4[2(s+1/2)+1] + 4[2(s-1/2)+1] = (2s+1)8_F \tag{11}$$

Thus, these multiplets contain $(2s+1)(8_B - 8_F)$ complex charged states with maximal spin $s_{hws} = s+1$.

An analogous analysis applies to higher spin 1/2 BPS multiplets in $\mathcal{N}=2$ compactifications such as (freely acting) orbifolds. For details see appendix A.

For some purposes, it is convenient to switch from the $\mathcal{N}=4$ notation in $D=4$ to an $\mathcal{N}=(1,0)$ notation in $D=10$.

In $D=4$, the little group for massive particles is $SO(3)$, that has rank one and admits only totally symmetric tensors as irreducible representations. Young Tableaux have only one row. Equivalently working with $SO(4) \sim SU(2)_L \times SU(2)_R$ one has un-dotted and dotted spinor indices which are separately symmetrized. Two anti-symmetrized $SU(2)$ spinor indices indeed represent the singlet.

In $D=10$ allowed Young Tableaux may have up to four rows since $SO(9)$ has rank four and five anti-symmetrized vector indices are equivalent to four. Yet in covariant notation, based on $SO(9,1)$ with rank five, it is necessary to consider anti-symmetric tensors with up to 5 indices. Going from $D=10$ to $D=4$ another important phenomenon takes place. $\mathcal{N}=(1,0)$ super-multiplets in $D=10$ may decompose into several $\mathcal{N}=4$ super-multiplets in $D=4$. Indeed, even at the massless level, *i.e.* setting $P=(p_\mu,0)$, the $\mathcal{N}=(1,0)$ super-gravity multiplet in $D=10$ decomposes into the $\mathcal{N}=4$ super-gravity multiplet along with six vector super-multiplets in $D=4$. On the other hand, the $\mathcal{N}=(1,0)$ massless vector super-multiplet in $D=10$ produces a single $\mathcal{N}=4$ massless vector super-multiplet in $D=4$.

Something similar happens for BPS and non-BPS massive multiplets. Moreover, if some of the supersymmetries are broken upon compactification, some of the states are eliminated or projected out from the spectrum, and further decomposition of the super-multiplets takes place. We will illustrate this point in Appendix A.

2 Massive non BPS super-multiplets

For later purposes, let us also analyze non BPS states with various spins in the higher mass levels of the perturbative heterotic spectrum [28, 29, 17]. We relate the light-cone gauge counting, based on $SO(8)$ for the left-moving superstring and $SO(24)$ for the right-moving bosonic string, to the relevant little group representations and show how BRST invariance allows to gauge away the non-physical fields. We furthermore display the BRST invariant vertex operators for the first and second mass levels explicitly. Eventually one has to combine left- and right- movers imposing level matching [17].

2.1 The left-moving superstring sector

Let us address the counting of degrees of freedom for the first two massive levels of the superstring (in ten dimensions), and show how to rearrange the states into representations of $SO(9)$, the little group for massive states [36, 37, 38, 39]. Before turning to the massive levels, let us discuss the massless case. Due to its simplicity, this level serves as a good example in the study BRST invariance and identification of physical degrees of freedom. Later on we will apply the lesson to the massive levels.

2.1.1 Massless Level (Left Movers)

In the light-cone gauge, the only bosonic states are given by[2]

$$\psi^i_{-1/2}|0\rangle \longrightarrow 8 = \square \tag{12}$$

which correspond to a massless gauge boson in the open string sector. In 10-dimensional covariant notation the state takes the form

$$A_M \psi^M_{-1/2}|0\rangle \qquad\qquad \mathcal{V}_A(z) = A_M \, e^{-\varphi} \, c \, \psi^M e^{ikX} \tag{13}$$

where the latter is the corresponding un-integrated vertex operator in the (-1) superghost picture that include the c-ghost. The polarization vector A_M is subject to the BRST constraints, namely it has to satisfy $k^M A_M = 0$, $k^2 = 0$ which suggests that A_M have nine

[2]With some abuse of notation we denote by $i, j, .. = 1, ...8$ the transverse directions.

independent components. However, using the fact that the BRST operator is nilpotent, we can add to the massless vertex operator the following operator

$$\delta\mathcal{V}(z) = [Q_{BRST}\,,\mathcal{U}(z)]\ , \tag{14}$$

where Q_{BRST} is the BRST charge given by

$$Q_{BRST} = \oint \frac{dz}{2\pi i}\ e^{\varphi}\,\eta\,\psi_M\,\partial X^M + \ldots\ . \tag{15}$$

For the massless level $\mathcal{U}(z)$ takes the form

$$\mathcal{U}(z) = \Lambda\ e^{-2\varphi}\,c\,\partial\xi\,e^{ikX} \tag{16}$$

with $k^2 = 0$. This allows to gauge away one un-physical component, thus a massless gauge boson in the left-moving sector exhibits as expected 8 physical degrees of freedom in agreement with the light-cone gauge quantization.

2.1.2 First Massive Level (Left Movers)

At the first mass level, in the light-cone gauge, the bosonic states are[3]

$$\begin{aligned}
\psi^i_{-3/2}|0\rangle &\longrightarrow 8 = \ydiagram{1} \\
\alpha^{(i}_{-1}\psi^{j)}_{-1/2}|0\rangle &\longrightarrow 35 = \ydiagram{2} \\
\alpha^{[i}_{-1}\psi^{j]}_{-1/2}|0\rangle &\longrightarrow 28 = \ydiagram{1,1} \\
\psi^i_{-1/2}\psi^j_{-1/2}\psi^k_{-1/2}|0\rangle &\longrightarrow 56 = \ydiagram{1,1,1} \\
\delta_{ij}\ \alpha^i_{-1}\psi^j_{-1/2}|0\rangle &\longrightarrow 1 = \bullet\ \ ,
\end{aligned} \tag{17}$$

where as usual (..) and [..] denote symmetrization and anti-symmetrization of the indices. Moreover, the symmetrized states do not contain the trace, which is treated separately.

In 10-dimensional covariant notation the states are given by

$$\widehat{E}\,\eta_{MN}\,\alpha^M_{-1}\psi^N_{-1/2}|0\rangle \qquad \widehat{W}_M\,\psi^M_{-3/2}|0\rangle \qquad \widehat{B}_{[MN]}\,\alpha^{[M}_{-1}\psi^{N]}_{-1/2}|0\rangle$$
$$\widehat{H}_{(MN)}\,\alpha^{(M}_{-1}\psi^{N)}_{-1/2}|0\rangle \qquad \widehat{C}_{[LMN]}\,\psi^L_{-1/2}\psi^M_{-1/2}\psi^N_{-1/2}|0\rangle$$

[3]Remember that, in the NS sector, states are built out of the vacuum by the action of an odd number of fermionic excitations. States with even fermionic excitation numbers are eliminated by the GSO projection.

The un-integrated vertex operators corresponding to the above states are given by

$$\mathcal{V}^{(-1)}_{\widehat{E}} = \widehat{E}\ e^{-\varphi}\, c\, \eta_{AB}\, \partial X^A\, \psi^B e^{ipX} \tag{18}$$

$$\mathcal{V}^{(-1)}_{\widehat{W}} = \widehat{W}_M\ c\, e^{-\varphi}\, \partial\psi^M\, e^{ipX} \tag{19}$$

$$\mathcal{V}^{(-1)}_{\widehat{B}} = \widehat{B}_{[MN]}\ e^{-\varphi}\, c\, \partial X^{[M}\, \psi^{N]} e^{ipX} \tag{20}$$

$$\mathcal{V}^{(-1)}_{\widehat{H}} = \widehat{H}_{(MN)}\ e^{-\varphi}\, c\, \partial X^{(M}\, \psi^{N)}\, e^{ipX} \tag{21}$$

$$\mathcal{V}^{(-1)}_{\widehat{C}} = \widehat{C}_{[MNP]}\ e^{-\phi}\, c\, \psi^{[M}\, \psi^N\, \psi^{P]}\, e^{ipX}\ . \tag{22}$$

Clearly, not all states correspond to physical degrees of freedom. BRST invariance allows one to gauge away the non-physical components. As for the massless level before, one can add an operator of the form

$$\delta\mathcal{V} = [Q_{BRST}\,, \mathcal{U} + \mathcal{U}']\ , \tag{23}$$

where, in contrast to the massless case, we distinguish between two different types of operators $\mathcal{U}$ and $\mathcal{U}'$. For the first massive level they take the form

$$\mathcal{U} = e^{-2\varphi}\, \partial\xi\, c\, \left[\lambda_{[AB]}\, \psi^A\, \psi^B + \omega_A\, \partial X^A\right] e^{ipX} \qquad \text{and} \qquad \mathcal{U}' = e^{-2\varphi}\, \partial^2\xi\, c\, \Lambda\, e^{ipX}\ , \tag{24}$$

with[4] $p^2 = -2$. This implies that BRST invariance allows us to gauge away one scalar, one vector and an anti-symmetric rank two tensor. Let us be more precise and display the effect of adding $[Q_{BRS}, \mathcal{U} + \mathcal{U}']$ to the polarization tensors *viz.*

$$\begin{aligned}
C_{[MNP]} &= \widehat{C}_{[MNP]} - ip_{[M}\lambda_{NP]} \\
H_{(MN)} &= \widehat{H}_{(MN)} - ip_{(M}\omega_{N)} \\
B_{[MN]} &= \widehat{B}_{[MN]} - \lambda_{[MN]} - ip_{[M}\omega_{N]} \\
W_M &= \widehat{W}_M - \omega_M - 2i\Lambda p_M \\
E &= \widehat{E} + 2\Lambda\ .
\end{aligned}$$

As previously discussed, BRST invariance allows one to gauge away $\widehat{B}_{[MN]}$, $\widehat{W}_M$, $\widehat{E}$. However, the structure of the gauging is quite involved. One is left with a totally anti-symmetric rank 3 tensor and a symmetric rank 2 tensor, which is in complete agreement with table 1. There we display the relation between the $SO(8)$ light-cone gauge group structure and the little group $SO(9)$ structure. For the first massive level we see that the $SO(8)$ covariant states in equation (17) can be rearranged into an anti-symmetric rank 3 and a symmetric rank 2 tensors invariant under the little group $SO(9)$.

Finally, we display the un-integrated physical vertex operators. In the (-1)-ghost picture they are given by [40, 41, 10, 11]

$$\mathcal{V}^{(-1)}_{C_{[MNP]}} = C_{[MNP]}\ e^{-\varphi}\, c\, \psi^M\, \psi^N\, \psi^P\, e^{ipX} \tag{25}$$

[4]Unless otherwise stated, we set $\alpha' = 2$ henceforth.

d.o.f	$SO(9)$	$SO(8)$
84	$\yng(1,1,1)$	$\yng(1,1,1)+\yng(1,1)$
44	$\yng(2)$	$\bullet+\yng(1)+\yng(2)$

Table 1: Decomposition of $SO(9)$ into $SO(8)$ representations for the first massive level.

and

$$\mathcal{V}^{(-1)}_{H_{(MN)}} = H_{(MN)}\; e^{-\varphi}\, c\,\partial X^{(M}\,\psi^{N)}\, e^{ipX}\ . \tag{26}$$

BRST invariance furthermore requires

$$p^M\, C_{[MNL]} = 0 \qquad p^M\, H_{(MN)} = 0 \qquad \eta^{MN}\, H_{(MN)} = 0\ . \tag{27}$$

in addition to the mass-shell condition $p^2 = -2 = -M^2$.

2.1.3 Second Massive Level (Left Movers)

At the second massive level the number of states increases drastically. Once again we display all states that arise from the canonical quantization performed in the light-cone gauge[5]

$$\begin{aligned}
\psi^i_{-5/2}|0\rangle &\longrightarrow 8 = \yng(1)\\
\alpha^i_{-1}\psi^j_{-3/2}|0\rangle &\longrightarrow 64 = \bullet+\yng(2)+\yng(1,1)\\
\alpha^i_{-2}\psi^j_{-1/2}|0\rangle &\longrightarrow 64 = \bullet+\yng(2)+\yng(1,1)\\
\psi^i_{-3/2}\psi^j_{-1/2}\psi^k_{-1/2}|0\rangle &\longrightarrow 224 = \yng(1)+\yng(1,1,1)+\yng(2,1)\\
\alpha^i_{-1}\alpha^j_{-1}\psi^k_{-1/2}|0\rangle &\longrightarrow 288 = \yng(1)+\yng(3)+\yng(2,1)\\
\alpha^i_{-1}\psi^j_{-1/2}\psi^k_{-1/2}\psi^l_{-1/2}|0\rangle &\longrightarrow 448 = \yng(1,1)+\yng(1,1,1,1)+\yng(2,1,1)\\
\psi^i_{-1/2}\psi^j_{-1/2}\psi^k_{-1/2}\psi^l_{-1/2}\psi^m_{-1/2}|0\rangle &\longrightarrow 56 = \yng(1,1,1,1,1)
\end{aligned}$$

[5]Note that in contrast to the first massive level we do not display every single state separately but rather in compact form. For instance the state $\alpha^i_{-1}\psi^j_{-3/2}|0\rangle$ contains a scalar, a symmetric and anti-symmetric rank 2 tensor.

In 10-dimensional covariant notation they take the form

$$\widehat{X}_{[MNPQR]}\ \psi^M_{-1/2}\psi^N_{-1/2}\psi^P_{-1/2}\psi^Q_{-1/2}\psi^R_{-1/2}|0\rangle$$

$$\widehat{Y}_{(M[N)PQ]}\ \alpha^M_{-1}\psi^N_{-1/2}\psi^P_{-1/2}\psi^Q_{-1/2}|0\rangle$$

$$\widehat{A}_{[MNPQ]}\ \alpha^M_{-1}\psi^N_{-1/2}\psi^P_{-1/2}\psi^Q_{-1/2}|0\rangle$$

$$\widehat{Z}_{(MNP)}\ \alpha^M_{-1}\alpha^N_{-1}\psi^P_{-1/2}|0\rangle$$

$$\widehat{B}_{[MNP]}\ \psi^M_{-1/2}\psi^N_{-1/2}\psi^P_{-3/2}|0\rangle$$

$$\widehat{V}_{(M[N)P]}\ \psi^M_{-1/2}\psi^N_{-1/2}\psi^P_{-3/2}|0\rangle \qquad \widehat{K}_{(M[N)P]}\ \alpha^M_{-1}\alpha^N_{-1}\psi^P_{-1/2}|0\rangle$$

$$\widehat{M}_{(MN)}\alpha^{(M}_{-1}\psi^{N)}_{-3/2}|0\rangle \qquad \widehat{L}_{(MN)}\alpha^{(M}_{-2}\psi^{N)}_{-1/2}|0\rangle$$

$$\widehat{D}_{[MN]}\ \eta_{KL}\alpha^L_{-1}\psi^K_{-1/2}\psi^M_{-1/2}\psi^N_{-1/2}|0\rangle \qquad \widehat{F}_{[MN]}\alpha^{[M}_{-1}\psi^{N]}_{-3/2}|0\rangle$$

$$\widehat{G}_{[MN]}\alpha^{[M}_{-2}\psi^{N]}_{-1/2}|0\rangle$$

$$\square \qquad \widehat{R}_M\ \eta_{AB}\psi^A_{-3/2}\psi^B_{-1/2}\psi^M_{-1/2}|0\rangle \qquad \widehat{S}_M\ \eta_{AB}\alpha^A_{-1}\alpha^B_{-1}\psi^M_{-1/2}|0\rangle$$

$$\widehat{T}_M\ \eta_{AB}\psi^A_{-1/2}\alpha^B_{-1}\alpha^M_{-1}|0\rangle \qquad \widehat{U}_M\psi^M_{-5/2}|0\rangle$$

$$\bullet \qquad \widehat{P}\eta_{AB}\alpha^A_{-2}\psi^B_{-1/2}|0\rangle \qquad \widehat{E}\eta_{AB}\alpha^A_{-1}\psi^B_{-3/2}|0\rangle$$

Clearly, not all of these represent physical degrees of freedom. BRST invariance allows one to gauge away un-physical components. Although we do not display the corresponding vertex operators one can shift them by

$$\delta\mathcal{V} = [Q_{BRST}, \mathcal{U} + \mathcal{U}'] \tag{28}$$

with

$$\begin{aligned}\mathcal{U} = e^{-2\varphi}\,\partial\xi\, c\Big[&\alpha_{[MNPQ]}\ \psi^M\psi^N\psi^P\psi^Q + \beta_{[MNP]}\ \psi^M\psi^N\partial X^P + \gamma_{[M(N]P)}\ \psi^M\psi^{(N}\partial X^{P)} \\ &+ \epsilon_{(MN)}\ \psi^{(M}\partial\psi^{N)} + \kappa_{(MN)}\ \partial X^M\partial X^N + \lambda_{[MN]}\ \psi^{[M}\psi^{N]} + \theta_M\ \partial^2 X^M \\ &+ \nu_M\ \psi^M\eta_{KL}\psi^K\partial X^L + \phi\ \eta_{AB}\partial X^A\partial X^B + \tau\ \eta_{AB}\psi^A\partial\psi^B\Big]e^{ipX}\end{aligned} \tag{29}$$

and

$$\mathcal{U}' = e^{-2\varphi}\,\partial^2\xi\, c\left[\sigma_{[MN]}\,\psi^M\,\psi^N + \rho_M\partial X^M\right]e^{ipX} \tag{30}$$

with $p^2 = -4$ in both cases. This allows to gauge away 2 scalars $\bullet$, 3 vectors $\square$, 2 symmetric rank 2 tensors, 2 anti-symmetric rank 2 tensors, 1 hooked Yang diagram, 1 totally antisymmetric rank 3 tensor, and 1 totally antisymmetric rank 4 tensor

d.o.f	$SO(9)$	$SO(8)$
126	$\ydiagram{1,1,1,1,1}$	$\ydiagram{1,1,1,1,1} + \ydiagram{1,1,1,1}$
156	$\ydiagram{3}$	$\bullet + \ydiagram{1} + \ydiagram{2} + \ydiagram{3}$
594	$\ydiagram{2,1,1}$	$\ydiagram{1,1} + \ydiagram{1,1,1} + \ydiagram{2,1} + \ydiagram{2,1,1}$
231	$\ydiagram{2,1}$	$\ydiagram{1} + \ydiagram{1,1} + \ydiagram{2} + \ydiagram{2,1}$
36	$\ydiagram{1,1}$	$\ydiagram{1} + \ydiagram{1,1}$
9	$\ydiagram{1}$	$\bullet + \ydiagram{1}$

Table 2: Decomposition of $SO(9)$ into $SO(8)$ representations for the second massive level.

$\ydiagram{1,1,1}$. Thus the truly physical degrees of freedom are represented by

$$\ydiagram{1,1,1,1,1} \qquad \ydiagram{2,1,1} \qquad \ydiagram{3} \qquad \ydiagram{2,1} \qquad \ydiagram{1,1} \qquad \ydiagram{1} \quad , \tag{31}$$

which is again in complete agreement with the group theoretical analysis. In Table 2 we display how the light-cone $SO(8)$ representatives can be rearranged into representations of the little group $SO(9)$. We find exactly the same degrees of freedom as displayed in equation (31). Their physical vertex operators are given by

$$\begin{aligned}
\mathcal{V}_X^{(-1)} &= X_{[MNPQR]}\; e^{-\varphi}\, c\, \psi^M\, \psi^N\, \psi^P\, \psi^Q\, \psi^R\, e^{ipX} \\
\mathcal{V}_Y^{(-1)} &= Y_{(M[N)PQ]}\; e^{-\varphi}\, c\, \partial X^M\, \psi^N\, \psi^P\, \psi^Q\, e^{ipX} \\
\mathcal{V}_Z^{(-1)} &= Z_{(MNP)}\; e^{-\varphi}\, c\, \partial X^M\, \partial X^N\, \psi^P\, e^{ipX} \\
\mathcal{V}_U^{(-1)} &= U_{(M[N)P]}\; e^{-\varphi}\, c\left[\partial\psi^M\, \psi^N\, \psi^P - 2\,\partial X^M\, \partial X^N\, \psi^P\right] e^{ipX} \\
\mathcal{V}_V^{(-1)} &= V_{[MN]}\; e^{-\varphi}\, c\, \left[\psi^M\, \psi^N\, \eta^{\perp}_{KL}\, \partial X^K\, \psi^L - \frac{7}{2}\,\psi^M\, \partial^2 X^N - 7\,\partial\psi^M\, \partial X^N\right]\; e^{ipX} \\
\mathcal{V}_W^{(-1)} &= W_M\; e^{-\varphi}\, c\left[\partial X^M\, \eta^{\perp}_{KL}\, \psi^K\, \partial X^L - 5\,\psi^M\, \eta^{\perp}_{KL}\, \partial X^K\, \partial X^L + 11\,\psi^M\, \eta^{\perp}_{KL}\, \psi^K\, \partial\psi^L\right] e^{ipX} \quad .
\end{aligned}$$

Here $\eta^{\perp}_{MN}$ is the $SO(9)$ invariant metric

$$\eta^{\perp}_{MN} = \eta_{MN} - \frac{p^M p^N}{p^2} = \eta_{MN} + \frac{p^M p^N}{M^2} \ . \tag{32}$$

In addition to the mass-shell condition $p^2 = -4 = -M^2$, BRST invariance implies the conditions

$$p^M X_{[MNPQR]} = p^M Y_{(M[N)PQ]} = p^M Z_{(MNP)} = p^M U_{(M[N)P]} = p^M V_{[MN]} = p^M W_M = 0 \tag{33}$$

and

$$\eta^{MN} Y_{(M[N)PQ]} = \eta^{MN} Z_{(MNP)} = \eta^{MN} U_{(M[N)P]} = 0 \ . \tag{34}$$

which makes the polarization tensors manifestly $SO(9)$ covariant.

With significant more effort, one can determine the physical vertex operators for bosonic states at higher mass levels. This is beyond the scope of our present analysis. For a decomposition into $SO(9)$ representations see [36, 37, 38, 39]. Let us now turn to the right-moving sector.

2.2 The right-moving bosonic string sector

The right-moving sector of the heterotic string consists of the 26-dimensional bosonic string [28, 29]. In contrast to the superstring, there are only bosonic excitations of the ground state. For simplicity and clarity, we will only consider the Cartan generators associated to the 16 purely right-moving directions. The generalization to the non-abelian case, *i.e.* including states with non-zero $\mathbf{p}_R$, is straight-forward but slightly more involved since after compactification their masses are moduli-dependent and may lead to gauge symmetry enhancement.

As for the superstring above, we will start with the massless case and then turn to the massive levels.

2.2.1 Massless level (Right movers)

The only massless excitations for the bosonic sector[6] are

$$\overline{\alpha}^a_{-1}|0\rangle \longrightarrow 24 = \square \tag{35}$$

[6] The tachyonic state $|0\rangle$ which is also present in the bosonic string does not survive in the heterotic state, due to the level matching condition.

where a runs over the 24 dimensional light-cone coordinates and describes a massless gauge boson in 26 dimensions. In 'covariant'[7] (in $D = 26$!!) notation the state and vertex operator are given by

$$B_A \overline{\alpha}^A_{-1}|0\rangle \qquad\qquad \mathcal{V}_B = B_A\ \overline{c}\,\overline{\partial} X^A\, e^{iKX}\ . \tag{36}$$

Here A runs over all 26 dimensions and BRST invariance enforces $K_A B^A = 0$ and $K^2 = k^2_{10d} + p^2_R = 0$. Furthermore the vertex operator is unique up to the addition of an operator of the form

$$\delta\mathcal{V} = [\overline{Q}_{BRST}\,, \overline{\mathcal{U}}] \tag{37}$$

where $\overline{Q}_{BRST}$ for the bosonic string takes the form

$$\overline{Q}_{BRST} = \oint \frac{d\overline{z}}{2\pi i}\ \frac{1}{2}\,\overline{c}\ \overline{\partial} X^A\, \overline{\partial} X_A(\overline{z}) + \ldots \tag{38}$$

and for the massless level $\overline{\mathcal{U}}$ is given by

$$\overline{\mathcal{U}} = \Lambda\, e^{iKX}\ , \tag{39}$$

which allows to gauge away one spurious state. Thus, taking into account the BRST constraint $K_A B^A = 0$, one has 24 pure physical degrees of freedom, in complete agreement with the number one obtains via light-cone gauge quantization.

2.2.2 First massive level (Right movers)

At the first mass level, in the light-cone gauge, the states are

$$\delta_{ab}\,\overline{\alpha}^a_{-1}\overline{\alpha}^b_{-1}|0\rangle \longrightarrow 1 = \bullet \tag{40}$$

$$\overline{\alpha}^a_{-2}|0\rangle \longrightarrow 24 = \square \tag{41}$$

$$\overline{\alpha}^{(a}_{-1}\overline{\alpha}^{b)}_{-1}|0\rangle \longrightarrow 299 = \square\!\square\ . \tag{42}$$

In 'covariant' 26-dimensional notation the states at the first mass level are given by

$$\widehat{E}\ \eta_{AB}\,\overline{\alpha}^A_{-1}\overline{\alpha}^B_{-1}|0\rangle \qquad\qquad \widehat{W}_A\ \overline{\alpha}^A_{-2}|0\rangle \qquad\qquad \widehat{H}_{(AB)}\ \overline{\alpha}^{(A}_{-1}\overline{\alpha}^{B)}_{-1}|0\rangle \tag{43}$$

and their corresponding vertex operators take the form

$$\mathcal{V}_{\widehat{E}} = \widehat{E}\ \overline{c}\,\eta_{AB}\,\overline{\partial} X^A \overline{\partial} X^B e^{ipX} \qquad\qquad \mathcal{V}_{\widehat{W}} = \widehat{W}_A\ \overline{c}\,\overline{\partial}^2 X^A e^{ipX}$$
$$V_{\widehat{H}} = \widehat{H}_{(AB)}\ \overline{c}\,\overline{\partial} X^{(A} \overline{\partial} X^{B)} e^{ipX}\ .$$

[7] We put *covariant* in quotes since only 10 of the 26 R-moving bosonic coordinates have L-moving counterparts.

As for the superstring before, not all components correspond to physical degrees of freedom. BRST invariance again allows one to gauge away the non-physical components. We can add an operator of the form

$$\delta\mathcal{V} = \left[\overline{Q}_{BRST}\, , \overline{\mathcal{U}} + \overline{\mathcal{U}}'\right] \tag{44}$$

with

$$\overline{\mathcal{U}} = \Gamma_A\, \overline{\partial} X^A e^{ipX} \qquad\qquad \overline{\mathcal{U}}' = \Lambda\, \overline{b}\,\overline{c}\, e^{ipX} \tag{45}$$

where $p^2 = -2$. Thus we can gauge away the massive scalar $\widehat{E}$ and the massive vector $\widehat{W}_A$ and we are left with a massive symmetric rank two tensor. This symmetric rank two tensor $H_{(AB)}$ accounts for the 324 degrees of freedom obtained in the light-cone gauge quantization. In Table 3 we display the decomposition of the SO(25) symmetric rank two tensor into the $SO(24)$ representation. One sees that H_{AB} indeed contains one $SO(24)$ symmetric rank two tensor, one $SO(24)$ vector and one $SO(24)$ scalar.

d.o.f	$SO(25)$	$SO(24)$
324	$\square\square$	$\bullet + \square + \square\square$

Table 3: Decomposition of $SO(25)$ into $SO(24)$ representations for the first massive level.

Below we display the vertex operator for the symmetric rank two tensor

$$\mathcal{V}_H(\overline{z}) = H_{(AB)}\ \overline{c}\,\overline{\partial} X^A\, \overline{\partial} X^B\, e^{ipX}\ . \tag{46}$$

The BRST conditions read

$$p^A\ H_{(AB)} = 0 \qquad\qquad \eta^{AB}\ H_{(AB)} = 0 \tag{47}$$

which reveals the $SO(25)$ invariance of the massive states $H_{(AB)}$, in addition to $p^2 = -2 = -M^2$.

2.2.3 Second mass level (Right movers)

In light-cone gauge the second mass level contains the following states

$$\overline{\alpha}^a_{-3}|0\rangle \longrightarrow 24 = \square \tag{48}$$

$$\overline{\alpha}^a_{-1}\overline{\alpha}^b_{-2}|0\rangle \longrightarrow 576 = \bullet + \begin{smallmatrix}\square\\ \square\end{smallmatrix} + \square\square \tag{49}$$

$$\overline{\alpha}^a_{-1}\overline{\alpha}^b_{-1}\overline{\alpha}^c_{-1}|0\rangle \longrightarrow 2600 = \square + \square\square\square\ . \tag{50}$$

In 'covariant' 26-dimensional form the relevant states are

$$\widehat{E}\ \eta_{AB}\overline{\alpha}^A_{-1}\overline{\alpha}^B_{-2}|0\rangle \qquad \widehat{W}_A\overline{\alpha}^A_{-3}|0\rangle \qquad \widehat{Y}_A\ \eta_{BC}\overline{\alpha}^A_{-1}\overline{\alpha}^B_{-1}\overline{\alpha}^C_{-1}|0\rangle \tag{51}$$

$$\widehat{B}_{[AB]}\ \overline{\alpha}^{[A}_{-1}\overline{\alpha}^{B]}_{-2}|0\rangle \qquad \widehat{H}_{(AB)}\ \overline{\alpha}^{(A}_{-1}\overline{\alpha}^{B)}_{-2}|0\rangle \qquad \widehat{S}_{(ABC)}\ \overline{\alpha}^A_{-1}\overline{\alpha}^B_{-1}\overline{\alpha}^C_{-1}|0\rangle\ , \tag{52}$$

where not all states correspond to pure physical degrees of freedom. Their corresponding vertex operators take the form

$$\begin{aligned}
\mathcal{V}_{\widehat{E}} &= \widehat{E}\ \overline{c}\,\eta_{AB}\,\overline{\partial}X^A\,\overline{\partial}^2X^B\,e^{ipX}\\
\mathcal{V}_{\widehat{W}} &= \widehat{W}_A\ \overline{c}\,\overline{\partial}^3X^A\,e^{ipX}\\
\mathcal{V}_{\widehat{Y}} &= \widehat{Y}_A\ \overline{c}\,\eta_{BC}\,\overline{\partial}X^A\,\overline{\partial}X^B\,\overline{\partial}X^C\,e^{ipX}\\
\mathcal{V}_{\widehat{B}} &= \widehat{B}_{[AB]}\ \overline{c}\,\overline{\partial}X^{[A}\,\overline{\partial}^2X^{B]}\,e^{ipX}\\
\mathcal{V}_{\widehat{H}} &= \widehat{H}_{(AB)}\ \overline{c}\,\overline{\partial}X^{(A}\,\overline{\partial}^2X^{B)}\,e^{ipX}\\
\mathcal{V}_{\widehat{C}} &= \widehat{S}_{(ABC)}\ \overline{c}\,\overline{\partial}X^A\,\overline{\partial}X^B\,\overline{\partial}X^C\,e^{ipX}
\end{aligned}$$

and analogously to the previous case BRST invariance allows us to add operators of the type $\left[Q_{BRST},\overline{\mathcal{U}}+\overline{\mathcal{U}}'\right]$ with

$$\overline{\mathcal{U}} = \left[\Lambda\ \eta_{AB}\,\overline{\partial}X^A\,\overline{\partial}X^B + \alpha_A\ \overline{\partial}^2X^A + \beta_{(AB)}\ \partial X^A\,\partial X^B\right]e^{ipX} \tag{53}$$

$$\overline{\mathcal{U}}' = \gamma_A\ \overline{b}\,\overline{c}\,\overline{\partial}X^A\,e^{ipX} \tag{54}$$

where $p^2 = -4$. Thus the truly physical degrees of freedom are just the antisymmetric

d.o.f	$SO(25)$	$SO(24)$
2900	$\Box\Box\Box$	$\bullet + \Box + \Box\Box + \Box\Box\Box$
300	$\begin{array}{c}\Box\\\Box\end{array}$	$\Box + \begin{array}{c}\Box\\\Box\end{array}$

Table 4: Decomposition of $SO(25)$ into $SO(24)$ representations for the second massive level.

rank 2 tensor and the completely symmetric rank 3 tensor.

In Table 4 we display how the $SO(25)$ massive states are decomposed into $SO(24)$ light-cone representations. Note that the antisymmetric rank 2 and the completely symmetric rank 3 tensors account for the states obtained via the light-cone gauge quantization. Below we display the vertex operators

$$\mathcal{V}_B = B_{[AB]}\ \overline{c}\,\overline{\partial}X^{[A}\,\overline{\partial}^2X^{B]}\,e^{ipX} \qquad \mathcal{V}_S = S_{(ABC)}\ \overline{c}\,\overline{\partial}X^A\,\overline{\partial}X^B\,\overline{\partial}X^C\,e^{ipX} \tag{55}$$

which have to satisfy the mass-shell condition $p^2 = -4 = -M^2$ and the BRST constraints

$$p^A B_{[AB]} = p^A S_{(ABC)} = 0 \qquad\qquad \eta^{AB} S_{(ABC)} = 0 \ . \tag{56}$$

The latter makes the polarization tensors manifestly $SO(25)$ covariant.

2.3 Heterotic string, Type I and Type II superstring

Though straightforward, proceeding to higher levels becomes more and more cumbersome, except possibly for the highest spin states (*i.e.* the leading Regge trajectory of the graviton). In a recent paper [39] (see also [36, 37, 38]), the massive spectrum of Heterotic, Type II, and Type I superstrings has been assembled into $SO(9)$ representations. On the other hand, production of open string Regge resonances in Type I and related models with open and unoriented strings has been studied in [5, 6, 7, 8, 9, 10, 11]. Given the expected duality between Heterotic and Type I strings, one may wonder if there is any way to compare HS states in the two descriptions. Clearly non BPS states are unstable and can decay into lower spin states [3, 4]. Yet, 1/2 BPS states should match on the two sides. The Type I counterpart of 1/2 BPS HS states should be HS excitations of wrapped D-strings. Indeed Heterotic / Type I duality has been carefully tested at the level of the BPS spectra both in toroidal compactifications with or without tensor structure and in other pairs (*e.g.* freely acting orbifolds etc). The relevant helicity super-trace ('topological index' [42]) does not allow to identify the spin unambiguously as we will momentarily see. Despite this, one can explore dynamical properties of perturbatively stable HS in Type I or Type II theories using methods similar to those exploited in [43].

3 Tri-linear couplings of charged higher spin BH's

Let us now discuss tri-linear couplings of higher spin states[8].

We will first consider 1/2 BPS states. In the NS sector vertex operators are of the form

$$V^{(-1)}_{1/2BPS} = A_M \psi^M e^{-\varphi} e^{iPX} V_R \tag{57}$$

where $\psi^M = (\psi^\mu, \psi^i)$ and V_R accounts for the Right-movers.

[8]For recent work on this issue from a different vantage point, see *e.g.* [44, 45].

3.1 Left-Movers (Superstring)

Let us focus on the Left-moving part first. BRST invariance imposes

$$P^2 = 0 = p^2 + |\mathbf{p_L}|^\mathbf{2} \qquad \mathbf{P^M A_M = p^\mu a_\mu + p^i_L \Phi_i = 0} \tag{58}$$

i.e. the Left-moving part is exactly identical to the Left-moving part of a massless vertex in $D = 10$. (Super)ghost charge violation requires to consider

$$G_L(z_1, z_2, z_3) = \langle c V_L^{(-1)}(z_1) c V_L^{(0)}(z_2) c V_L^{(-1)}(z_3) \rangle \tag{59}$$

where

$$V_L^{(0)}(z) = A_M(\partial X^M + iP\psi\psi^M)e^{iPX} \tag{60}$$

Performing the contractions one gets

$$G_L(z_1, z_2, z_3) = [A_1(P_2 - P_3)A_2A_3 + A_2(P_3 - P_1)A_3A_1 + A_3(P_1 - P_2)A_1A_2]\delta(P_1 + P_2 + P_3) \tag{61}$$

that is independent of z_i, totally anti-symmetric and vanishing on-shell for real momenta.

As a result there is no physical (decay) process involving three 1/2 BPS states at tree level! This is largely a consequence of the kinematics and the absence of quantum corrections to the mass and charge and is independent of the spin of the states which is mostly contributed by the right-moving bosonic string excitations. We thus expect this non-renormalization property to hold to all orders in perturbation theory and even non-perturbatively. This seems to raise a puzzle in the microscopic counting of states that reproduce the degeneracy of small 1/2 BPS Black Holes [15, 16]. Indeed (extended) supersymmetry in $D = 4$ is only compatible with zero horizon rotation (horizon spin), spin can only emerge from hair d.o.f. *i.e.* from the global supersymmetry parameter that carries spinor indices and vanishes at the horizon (susy enhancement!) while assuming constant value at infinity. HS (BPS) states are a peculiarity of String Theory and may require some reconsideration. While it is important to study the (thermo)dynamical properties of these HS states, one should keep in mind that by going from weak coupling ($g_s << 1$) where the (heterotic) string description is reliable to strong coupling ($g_s >> 1$) where the supergravity description should take over, non-perturbative effects may 'replace' HS supermultiplets with minimal spin ones *i.e.* vector multiplets. Indeed, the only 'protected' information is coded in the helicity supertace [42]

$$\mathcal{B}_4 = Tr(-)^{2h}(2h)^4 \tag{62}$$

and one can easily check that

$$\mathcal{B}^{1/2BPS}_{4,S} = (2S + 1)\mathcal{B}^{1/2BPS}_{4,S=0} \tag{63}$$

where S is the 'intermediate' spin in the super-multiplet with $(2S+1)(8_B+8_F)$ d.o.f. so that $S=0$ correspond to the 'standard'(massless or 1/2 BPS) vector multiplet. Yet in perturbation theory we don't see any trace of the 'instability' of these HS states ...

The situation drastically changes if one of the three states (say the one at z_2) is non BPS. In order to have a non-vanishing coupling with two 1/2 BPS states one should select the non-BPS state properly. For instance at the first massive level in the NS sector one has two kinds of physical vertex operators

$$V^{(0)}_{H,L} = H_{MN}[\partial X^M \partial X^N + \frac{i}{2} P\psi(\psi^M \partial X^M + \psi^N \partial X^M) + \frac{1}{2}(\psi^M \partial \psi^M + \psi^N \partial \psi^M)] e^{iPX} \quad (64)$$

and

$$V_C = C_{LMN}(\partial X^L + iP\psi\psi^L)\psi^M \psi^N e^{iPX} \ . \quad (65)$$

BRST symmetry implies

$$H_{MN} = H_{MN} \qquad \eta^{MN} H_{MN} = 0 \qquad P^M H_{MN} = 0 \quad (66)$$

leading to a massive spin 2 particle with 44 physical polarizations and

$$C_{LMN} = -C_{MLN} = -C_{LNM} = -C_{NML} \qquad P^M C_{LMN} = 0 \quad (67)$$

leading to a massive 3-form with 84 physical polarizations.

The latter couples to two 1/2 BPS states through the symmetric combination

$$(P_1 - P_2)^L A_1^M A_2^N C_{LMN} \quad (68)$$

which is gauge invariant under $A_i \to A_i + \xi P_i$ thanks to momentum conservation and transversality of C_{LMN}.

Similarly, the non-vanishing gauge-invariant coupling to the spin 2 combination reads

$$H_{MN} F_1^{ML} F_{2L}^N \ , \quad (69)$$

where

$$F_{MN} = P_M A_N - P_N A_M \quad (70)$$

thus exposing the manifest gauge invariance wrt $A_i \to A_i + \xi P_i$.

In order to determine the on-shell coupling of two bosonic 1/2 BPS vertex operators to higher mass non BPS states it proves convenient to start from the OPE of two fermionic vertex operators. The latter can be both taken to be in the canonical (-1/2) picture and produce bosons in the canonical (-1) picture. Expanding wrt the middle point [46] allows one to extract the on-shell coupling to higher mass states. $\mathcal{N}=4$ supersymmetry

transformations determine the coupling between two bosonic states. The relevant OPE of two fermionic 1/2 BPS vertex operators reads

$$\begin{aligned} e^{-\varphi/2}S^{A_1}e^{iP_1X}(z_1)e^{-\varphi/2}S^{A_2}e^{iP_2X}(z_2) \approx \\ z_{12}^{P_1P_2-1}\ e^{-\varphi}\,(1+...)\ e^{i(P_1+P_2)X}(1+z_{12}(P_1-P_2)\partial X...) \\ \times\,(\Gamma_M^{A_1A_2}\psi^M + z_{12}\Gamma_{MNL}^{A_1A_2}\psi^M\psi^N\psi^L + z_{12}^2\Gamma_{M_1...M_5}^{A_1A_2}\psi^{M_1}...\psi^{M_5}+...) \end{aligned} \tag{71}$$

where dots stand for derivatives.

Let us take a closer look at the coupling of two BPS fermions to the first massive excitations, $C_{[LMN]}$ and $H_{(MN)}$. The antisymmetric rank 3 tensor $C_{[LMN]}$ will couple through $\Gamma_{MNL}^{A_1A_2}$ in (71). On the other hand $H_{(MN)}$ will couple through $(P_1-P_2)_{(N}\Gamma_{M)}^{A_1A_2}$. Note that in contrast to the coupling of three 1/2 BPS states the coupling of two 1/2 BPS states to a non-BPS state at the first mass level is (graded) symmetric. At next level one has a coupling through $\Gamma_{M_1...M_5}^{A_1A_2}$ which is antisymmetric.

Generalizing this statement to higher mass levels, one observes that the coupling of a massive non-BPS vertex operator at odd (even) level to two 1/2 BPS vertex operators is (anti) symmetric under the exchange of the latter two.

3.2 Right-Movers (Bosonic String)

In the Heterotic string one has to consider the contribution of the right-moving sector to the 3-point amplitudes. The relevant correlation function on the world-sheet is of the form

$$G_R(\bar z_1,\bar z_2,\bar z_3) = \langle \bar c H_1[\bar\partial^{k_i}X_R^{A_i}]e^{iP_{1R}X_R}(\bar z_1)\bar c H_2[\bar\partial^{m_j}X_R^{B_j}]e^{iP_{2R}X_R}(\bar z_2)\bar c H_3[\bar\partial^{n_k}X_R^{C_k}]e^{iP_{3R}X_R}(\bar z_3)\rangle \tag{72}$$

where

$$H[\bar\partial^{k_i}X_R^{A_i}] = H_{A_1...A_n}\bar\partial^{k_1}X^{A_1}...\bar\partial^{k_n}X^{A_n} = H_{A_1...A_n}\left[\frac{\partial}{\partial\beta_{A_1}^{(k_1)}}\cdots\frac{\partial}{\partial\beta_{A_n}^{(k_n)}}\exp\sum_k \beta_A^{(k)}\bar\partial^k X^A\right]_{\beta_A^{(k)}=0} \tag{73}$$

so that

$$G_R(\bar z_1,\bar z_2,\bar z_3) = \bar c(\bar z_1)\bar c(\bar z_2)\bar c(\bar z_3)H^1_{...A_i...}\cdots\frac{\partial}{\partial\beta_{A_i}^{(k_i)}}...H^2_{...B_j...}\cdots\frac{\partial}{\partial\beta_{B_j}^{(m_j)}}...H^3_{...C_k...}\cdots\frac{\partial}{\partial\beta_{C_k}^{(n_k)}}...\mathcal{W}|_{\beta_A^{(k)}=0} \tag{74}$$

where

$$\mathcal{W} = \prod_{i<j}\bar z_{ij}^{P_iP_j}\exp i\sum_{i\neq j}(-)^{k_j}k_j!\frac{P_i\beta_j^{(k_j)}}{\bar z_{ij}^{k_j+1}}\exp\sum_{i<j}(-)^{l_i+k_j}(l_i+k_j)!\frac{\beta_i^{(l_i)}\beta_j^{(k_j)}}{\bar z_{ij}^{k_j+l_i}}\ . \tag{75}$$

Except for the first Regge trajectory involving only $\bar{\partial} X^A$ *i.e.* all $k_n = 1$, the result is unwieldy. Consider three HS states with spin S_i, thus each containing S_i conformal fields $\bar{\partial} X^A$ in the vertex operator. Provided $\sum_i (S_i - T_i) = 2K$ (even number) with $T_i \leq S_i$, T_i indices can contract with the momenta P_j of the other insertions. The remaining ones should contract with one another. Denoting $S'_i = S_i - T_i$ then $S'_{12} = (S'_1 + S'_2 - S'_3)/2$ indices of H_1 will contract with H_2 and so on.

For totally symmetric tensors – not necessarily with maximal spin $S_R = N_R$, as in the first Regge trajectory – the relevant tri-linear coupling reads

$$\sum_{T_i \leq S_i : \sum_i (S_i - T_i) = 2K} (\alpha')^{T_1+T_2+T_3/2} \prod_i \binom{S_i}{T_i} \binom{S'_i}{S'_{i,i+1}} P_{23}^{A_1} \dots P_{23}^{A_{T_1}} H^1_{A_1 \dots A_{T_1}}{}^{D_1 \dots D_{S'_{31}}}{}_{F_1 \dots F_{S'_{12}}}$$
$$P_{31}^{B_1} \dots P_{31}^{B_{T_2}} H^2_{B_1 \dots B_{T_2}}{}^{F_1 \dots F_{S'_{12}}}{}_{E_1 \dots E_{S'_{23}}} P_{12}^{C_1} \dots P_{12}^{C_{T_3}} H^3_{C_1 \dots C_{T_3}}{}^{E_1 \dots E_{S'_{23}}}{}_{D_1 \dots D_{S'_{31}}} \quad (76)$$

When at least two vertex operators are identical (say 1 and 3), the result is (anti)symmetric under the exchange of the two depending of whether S_2 is even (or odd).

In the special case in which some non-abelian current algebra survives, at lowest level ($N_R = 1$) one has the currents J^a. At the next level $N_R = 2$ one finds the primary $H_a = d_{abc} J^b J^c$ where $d_{abc} = Tr(T_a\{T_b, T_c\})$ is totally symmetric and traceless wrt the Cartan-Killing metric in the absence of abelian ideals.

The resulting coupling to two currents is

$$\langle J^a(z) H^b(u) J^c(w) \rangle = \frac{d^{abc}}{(z-u)^2 (w-u)^2} \quad (77)$$

which is manifestly symmetric as expected.

It is amusing that a similar situation prevails in Type I and other models with open and unoriented strings even for non BPS HS states [5, 6, 7, 8, 9, 10, 11]. Indeed twist symmetry implies that levels differing by an odd integer have opposite symmetry under the exchange of the two ends of the (open unoriented) string.

4 Scattering Amplitudes, Form Factors and all that

In order to further explore the dynamical properties of BPS HS states, we will compute the 4-point scattering amplitudes involving states of this kind at tree level [25]. Due to central charge conservation we will need at least two such states in the process. It should also be clear that, thanks to mass and charge conservation, there is no physical decay amplitude of one massive BPS state to three (or more) states be either BPS or not. Indeed $M_{BPS} = |\mathbf{p_L}| = |\sum_\ell \mathbf{p_L}(\ell)| \leq \sum_\ell |\mathbf{p_L}(\ell)| \leq \sum_\ell \mathbf{M}(\ell)$ and the process is kinematically

allowed only when all momenta, including the internal components representing central charges, are aligned so that there is no true scattering.

For simplicity, we will focus on the 4-point amplitude containing two massless gauge bosons of the visible sector[9] and two higher spin fields that describe for instance the scattering of a 'photon' off a very massive HS object. We will analyze the pole structure and low energy limit and identify the exchanged particles. This will allow us to determine the coupling of such higher spin states to the massless particles such as the graviton and dilaton. Along the way we need the generating function for a correlator containing an arbitrary number of the conformal fields ∂X^A, whose derivation is presented in appendix C.

More specifically, we are interested in the 4-point amplitude

$$\mathcal{A}_{AA\to\mathcal{H}_i\bar{\mathcal{H}}_j} = \left\langle V^{(-1)}_{\mathcal{H}_i}(z_1)\, V^{(0)}_A(z_2)\, V^{(0)}_A(z_3)\, V^{(-1)}_{\mathcal{H}_j}(z_4)\right\rangle \tag{78}$$

with the vertex operators for the gauge fields and the higher spin fields

$$\begin{aligned} V^{(-1)}_{\mathcal{H}_i} &= \mathcal{H}_{i,\mu_1\cdots\mu_s} e^{-\varphi}\psi^i e^{i\mathbf{p}_L X}\bar{\partial}X^{\mu_1}\cdots\bar{\partial}X^{\mu_s} e^{i\mathbf{p}_R X} e^{ipX} &(79)\\ V^{(0)}_A &= a_\mu\left(\partial X^\mu - ik\cdot\psi\psi^\mu\right)\bar{J}^a e^{ikX}\ . &(80)\end{aligned}$$

As discussed in section 1, $\mathcal{H}_i$ accounts for 5 spin s states. The BRST conditions for the projections read

$$k^\mu a_\mu = p^{\mu_i}\,\mathcal{H}_{i,\mu_1\cdots\mu_s} = p^i_L\,\mathcal{H}_{i,\mu_1\cdots\mu_s} = \eta^{\mu_i\mu_j}\,\mathcal{H}_{i,\mu_1\cdots\mu_s} = 0 \qquad \forall i,j\neq 0 \tag{81}$$

and the mass is given by

$$M^2 = |\mathbf{p}|^2_L = |\mathbf{p}|^2_R + 2(s-1)\ . \tag{82}$$

As for the three-point functions discussed in the previous section, the amplitude splits into a holomorphic and an anti-holomorphic part. Decomposing the HS field $H_{i\mu_1\ldots\mu_s}$ into the tensor product of an internal vector v_i and a space-time tensor $h_{\mu_1\ldots\mu_s}$

$$H_{i\mu_1\ldots\mu_s} = v_i\otimes h_{\mu_1\ldots\mu_s}$$

the left-moving part yields [25]

$$\begin{aligned} \mathcal{W}^L(z_i) = v_{1i}\, a_{2\kappa}\, a_{3\lambda}\, v_{4j}\, &\left\langle e^{-\phi(z_1)}\ e^{-\phi(z_4)}\right\rangle\left\langle e^{i\mathbf{p}_L\mathbf{X}_L}(z_1)\, e^{-i\mathbf{p}_L\mathbf{X}_L}(z_4)\right\rangle\left\langle\psi^i(z_1)\psi^j(z_4)\right\rangle \quad (83)\\ &\left\langle\psi^{\mu_0}\, e^{ip_1X}(z_1)\,[\partial X^\kappa - i(k_2\cdot\psi)\psi^\kappa]\, e^{ik_2X}(z_2)\,\left[\partial X^\lambda - i(k_3\cdot\psi\right]\psi^\lambda) e^{ik_3X}(z_3)\psi^{\nu_0}\, e^{ip_4X}(z_4)\right\rangle\ .\end{aligned}$$

[9]Following [25] the visible sector denotes the gauge sector already present in 10 dimensions.

The right-moving part takes the form

$$\mathcal{W}^R(\bar z_i) = h_{1\mu_1\ldots\mu_s} h_{4\nu_1\ldots\nu_s} \Big\langle \bar J^{a_2}(\bar z_2) \bar J^{a_3}(\bar z_3) \Big\rangle \; \Big\langle \; e^{i\mathbf{p}_R^1 \mathbf{X}_R}(\bar z_1) e^{i\mathbf{p}_R^4 \mathbf{X}_R}(\bar z_4) \Big\rangle \\ \Big\langle \bar\partial X^{\mu_1} \cdots \bar\partial X^{\mu_s} e^{ip_1 X}(\bar z_1)\, e^{ik_2 X}(\bar z_2)\, e^{ik_3 X}(\bar z_3)\, \bar\partial X^{\nu_1} \cdots \bar\partial X^{\nu_s} e^{ip_4 X}(\bar z_4) \Big\rangle \; . \tag{84}$$

The details of the calculation are relegated to appendix B. After reinstating normalization factors, one finds

$$\mathcal{A} = -\frac{g^2_{_{YM}}}{M_s^2} (2\pi)^4 \delta(\Sigma_i p_i) \frac{1}{k_2 k_3} v_1 \cdot v_4 \, (p_4 f_2 f_3 p_4) h_{1\mu_1\ldots\mu_s} \, h_{4\nu_1\ldots\nu_s} \, \mathcal{F}_{SV} \, \mathcal{J}_R^{\mu_1\ldots\mu_s\nu_1\ldots\nu_s} \tag{85}$$

where $\mathcal{J}_R$ is given by equation (172) in appendix B, f_i denotes the linearized field strength

$$f_{i\mu\nu} = k_{i\mu} a_{i\nu} - a_{i\mu} k_{i\nu} \tag{86}$$

the contraction $(p_4 f_2 f_3 p_4)$ is understood as

$$p_{4_\mu} \, {f_2}^{\mu\nu} \, f_{3\nu\rho} \, {p_4}^\rho \tag{87}$$

and the Shapiro-Virasoro $\mathcal{F}_{SV}$ factor is given by (recall $\alpha' = 2$)

$$\mathcal{F}_{SV} = \frac{\Gamma(1 + k_2\, k_3)\Gamma(k_3\, p_4)\Gamma(k_3\, p_1)}{\Gamma(2 - k_2\, k_3)\Gamma(-k_3\, p_4)\Gamma(-k_3\, p_1)} = \frac{\Gamma\left(1 - \frac{t}{2}\right)\Gamma\left(\frac{M^2}{2} - \frac{s}{2}\right)\Gamma\left(\frac{M^2}{2} - \frac{u}{2}\right)}{\Gamma\left(2 + \frac{t}{2}\right)\Gamma\left(\frac{s}{2} - \frac{M^2}{2}\right)\Gamma\left(\frac{u}{2} - \frac{M^2}{2}\right)} \; , \tag{88}$$

where in the last step we used the usual definitions of Mandelstam variables

$$s = -(k_2 + p_1)^2 = -2k_2 p_1 + M^2 \tag{89}$$
$$t = -(k_2 + k_3)^2 = -2k_2 k_3 \tag{90}$$
$$u = -(k_3 + p_1)^2 = -2k_3 p_1 + M^2 \; . \tag{91}$$

Clearly, the amplitude is invariant under the gauge transformation

$$a_{r\mu} \longrightarrow a_{r\mu} + i\Lambda_r k_\mu \tag{92}$$

but also under a fake "gauge transformation"

$$v_{\ell M} \longrightarrow v_{\ell M} + i\Lambda_\ell P_{\ell M} \, . \tag{93}$$

Here $v^M = (0, v^i)$ is the ten-dimensional left-handed polarization vector and P_M denotes the whole ten-dimensional momentum. To expose this invariance let us perform such a transformation for v_1. The product $v_1 \cdot v_4$ gives $\mathbf{p}_L^1 \cdot v_4$, since v_4^M vanishes in space-time. Due to central charge conservation $\mathbf{p}_L^1 = -\mathbf{p}_L^4$ and with BRST invariance the term $\mathbf{p}_L^1 \cdot v_4$ vanishes. Analogously one can show gauge invariance of the amplitude under a transformation (93) for v_4. The reason for this "gauge" invariance of v_i lies in the fact that

the left-moving part of a BPS state vertex operator is exactly the one of a massless 10-dimensional gauge boson vertex operator. Although we will not exploit this fake "gauge invariance" any further in the present analysis, we would like to add that it might prove useful in the computation of amplitudes involving several BPS states.

Using momentum conservation and BRST invariance allows us to write the amplitude in a way that displays its symmetries, which are manifest in the form

$$\mathcal{A} = -\frac{g_{_{YM}}^2}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)\frac{v_1 \cdot v_4}{k_2 k_3}\,\left[\frac{1}{2}p_1(f_2 f_3 + f_3 f_2)p_4 + \frac{1}{4}k_2 \cdot k_3 (f_2 \cdot f_3)\right] \\ \times h_{1_{\mu_1 \ldots \mu_s}}\, h_{4_{\nu_1 \ldots \nu_s}}\, \mathcal{F}_{SV}\, \mathcal{J}_R^{\mu_1 \ldots \mu_s \nu_1 \ldots \nu_s} \tag{94}$$

where $(f_2 \cdot f_3) = f_{2\mu\nu} f_3{}^{\mu\nu}$. With $\mathcal{J}_R$, see eq. (172), being symmetric under the exchange of $1 \leftrightarrow 4$ as well as $2 \leftrightarrow 3$ in this from the amplitude reveals the expected symmetries under the exchange of $1 \leftrightarrow 4$ and $2 \leftrightarrow 3$.

4.1 Specific case $s = 2$

In order to further simplify the discussion, we will henceforth consider the specific case of $s = 2$. In this case one can display $\mathcal{J}_R$ explicitly and compactly. Then the coupling of such a spin 2 state to massless particles such as graviton, dilaton or antisymmetric rank two tensor can be extracted by looking at the residue of the t-channel pole $1/k_2 k_3$.

The amplitude for $s = 2$ takes the form

$$\mathcal{A} = -\frac{g_{_{YM}}^2}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)\frac{1}{k_2 k_3}\;\mathcal{F}_{SV}\;v_1 \cdot v_4\,(p_4 f_2 f_3 p_4) \times \left[L_0 + \frac{k_3\,p_4 + 1}{k_3 p_1}L_1 + \frac{k_3\,p_1 + 1}{k_3 p_4}L_2 \right. \\ \left. + \frac{(k_3 p_4 + 2)(k_3 p_4 + 1)}{(k_3 p_1)(k_3 p_1 - 1)}L_3 + \frac{(k_3 p_1 + 2)(k_3 p_1 + 1)}{(k_3 p_4)(k_3 p_4 - 1)}L_4\right] \tag{95}$$

with the L_i's given by

$$\begin{aligned} L_0 &= 2\,(h_1 \cdot h_4) - 4(k_2\,h_1\,h_4\,k_2) - 4(k_3\,h_1\,h_4\,k_3) \\ &\quad + (k_2 h_1 k_2)(k_2 h_4 k_2) + (k_3 h_1 k_3)(k_3 h_4 k_3) + 4(k_2 h_1 k_3)(k_2 h_4 k_3) \\ L_1 &= 4(k_3 h_1 h_4 k_2) - 2(k_2 h_1 k_3)(k_2 h_4 k_2) - 2(k_3 h_1 k_3)(k_3 h_4 k_2) \\ L_2 &= 4(k_2 h_1 h_4 k_3) - 2(k_2 h_1 k_2)(k_2 h_4 k_3) - 2(k_3 h_1 k_2)(k_3 h_4 k_3) \\ L_3 &= (k_3 h_1 k_3)(k_2 h_4 k_2) \\ L_4 &= (k_2 h_1 k_2)(k_3 h_4 k_3)\ . \end{aligned}$$

Let us now determine the residue of $1/k_2 k_3$ that describes the exchange of a massless particle, such as the graviton, the dilaton or the antisymmetric rank 2 tensor. The Shapiro-Virasoro factor gives 1 in the limit $k_2 k_3 \to 0$. Keeping further in mind that the

residue implies $k_2k_3 = 0$, which further implies via momentum conservation $k_2 = -k_3$ we obtain

$$\frac{2}{k_2k_3} v_1 \cdot v_4\,(p_4 f_2 f_3 p_4)(h_1 h_2) = \frac{2}{k_2k_3}(p_4 f_2 f_3 p_4)(\mathcal{H}_1\mathcal{H}_4)\ , \tag{96}$$

where we have reinstated the full projector $\mathcal{H}_{i\mu_1\mu_2} = v_i \otimes h_{\mu_1\mu_2}$.

As mentioned above the residue describes the exchange of a graviton, axion or dilaton. Note that the expression (96) does not contain any α' terms, they cancel among each other once one imposes $k_2 = -k_3$. That is somewhat expected since the on-shell coupling of the massless fields, graviton, axion and dilaton, to the gauge bosons as well as to the massive BPS states do not contain any α' corrections even for complex momenta.

In order to determine the couplings of HS states to a specific massless field such as the graviton, we choose the gauge bosons to have opposite helicity [11, 47]. In this case the exchanged massless state is the graviton since $T_{\mu\nu} = f^+_{\mu\rho} f^{-\rho}_\nu$ is symmetric and conserved. (Pseudo)scalars couple to $f^+_{\mu\nu} f^{+\mu\nu} \pm f^-_{\mu\nu} f^{-\mu\nu}$. For concreteness we choose positive helicity for $a_2 = a_2^+$ and negative helicity for $a_3 = a_3^-$. Then the residue (96) takes the form

$$\frac{2}{k_2k_3}(p_4 f_2^+ f_3^- p_1)(\mathcal{H}_1\mathcal{H}_4) = \frac{2}{k_2k_3}\left(f_2^+ f_3^-\right)_{\mu\nu}(p_1 - p_4)^\mu(\mathcal{H}_1\mathcal{H}_4)(p_1 - p_4)^\nu \tag{97}$$

This structure is of the expected type

$$T^{(2,3)}{}_{\mu\nu} T^{(1,4)\,\mu\nu} \tag{98}$$

where

$$T^{(2,3)}{}_{\mu\nu} = \left(f_2^+ f_3^-\right)_{\mu\nu} \qquad \text{and} \qquad T^{(1,4)\,\mu\nu} = (p_1 - p_4)^\mu(\mathcal{H}_1\mathcal{H}_4)(p_1 - p_4)^\nu \tag{99}$$

denote the stress energy tensor of two gauge fields and two higher spin fields, respectively.

In order to set the stage for the study of the low energy behavior and spin effects, it proves convenient to express the polarizations in terms of the momenta p_i and k_i. Symmetric tensor polarizations

$$h_1^{\mu\nu} = \frac{1}{\sqrt{2}}\left(w_2^\mu w_3^\nu + w_3^\mu w_2^\nu\right) \qquad \widetilde{h}_1^{\mu\nu} = \frac{1}{\sqrt{2}}\left(\widetilde{w}_2^\mu \widetilde{w}_3^\nu + \widetilde{w}_3^\mu \widetilde{w}_2^\nu\right) \tag{100}$$

$$h_2^{\mu\nu} = \frac{1}{\sqrt{2}}\left(w_3^\mu w_1^\nu + w_1^\mu w_3^\nu\right) \qquad \widetilde{h}_2^{\mu\nu} = \frac{1}{\sqrt{2}}\left(\widetilde{w}_3^\mu \widetilde{w}_1^\nu + \widetilde{w}_1^\mu \widetilde{w}_3^\nu\right) \tag{101}$$

$$h_3^{\mu\nu} = \frac{1}{\sqrt{2}}\left(w_1^\mu w_2^\nu + w_2^\mu w_1^\nu\right) \qquad \widetilde{h}_3^{\mu\nu} = \frac{1}{\sqrt{2}}\left(\widetilde{w}_1^\mu \widetilde{w}_2^\nu + \widetilde{w}_2^\mu \widetilde{w}_1^\nu\right) \tag{102}$$

$$h_4^{\mu\nu} = \frac{1}{\sqrt{2}}\left(w_1^\mu w_1^\nu - w_3^\mu w_3^\nu\right) \qquad \widetilde{h}_4^{\mu\nu} = \frac{1}{\sqrt{2}}\left(\widetilde{w}_1^\mu \widetilde{w}_1^\nu - \widetilde{w}_3^\mu \widetilde{w}_3^\nu\right) \tag{103}$$

$$h_5^{\mu\nu} = \frac{1}{\sqrt{6}}\left(w_1^\mu w_1^\nu - 2w_2^\mu w_2^\nu + w_3^\mu w_3^\nu\right) \qquad \widetilde{h}_5^{\mu\nu} = \frac{1}{\sqrt{6}}\left(\widetilde{w}_1^\mu \widetilde{w}_1^\nu - 2\widetilde{w}_2^\mu \widetilde{w}_2^\nu + \widetilde{w}_3^\mu \widetilde{w}_3^\nu\right). \tag{104}$$

can be constructed from massive spin 1 polarizations. The massive spin 1 polarizations w_i, transverse to p_1, are given by

$$w_1^\mu = \frac{1}{(k_2k_3)\sqrt{-2F-M^2}}\epsilon^{\mu\nu\rho\sigma}k_{2\nu}k_{3\rho}p_{4\sigma} \tag{105}$$

$$w_2^\mu = \frac{M}{(k_2p_1)}k_2'^\mu \tag{106}$$

$$w_3^\mu = \frac{(k_2p_1)}{(k_2k_3)\sqrt{-2F-M^2}}\left(k_3'^\mu + \frac{(k_3'k_2')}{(k_2'k_2')}k_3'^\mu\right) \ , \tag{107}$$

where $k_i'^\mu = \left(k_i^\mu + \frac{(k_ip_1)}{M^2}p_1^\mu\right)$ and

$$F = \frac{(k_2p_1)(k_3p_1)}{k_2k_3} \ . \tag{108}$$

The polarizations $\widetilde{w}_i$, transverse to p_4, are given by $\widetilde{w}_1^\mu = w_1^\mu$ and

$$\widetilde{w}_2^\mu = \frac{M}{(k_2p_4)}k_2''^\mu \tag{109}$$

$$\widetilde{w}_3^\mu = \frac{(k_2p_4)}{(k_2k_3)\sqrt{-2F-M^2}}\left(k_3''^\mu + \frac{(k_3''k_2'')}{(k_2''k_2'')}k_3''^\mu\right) \tag{110}$$

with $k_i''^\mu = \left(k_i^\mu + \frac{(k_ip_4)}{M^2}p_4^\mu\right)$.

It is straightforward to show that w_i and $\widetilde{w}_i$ represent two ortho-normal bases for space-like vectors[10]. This holds true also for the above bases of symmetric massive spin 2 polarizations $h_i^{\mu\nu}$ and $\widetilde{h}_i^{\mu\nu}$.

Given the above choice of tensor polarizations, one can proceed analyzing specific amplitudes with fixed polarizations for the higher spin states. It turns out that the polarizations h_2, h_3 only couple to $\widetilde{h}_2$, $\widetilde{h}_3$. Here we focus on this subset. Moreover we have to specify the helicity for the two massless vector bosons, which can be written as [48]

$$a_2 = \frac{1}{\sqrt{2}}\left(n_1^\mu + i\lambda_2 n_2^\mu\right) \qquad\qquad a_3 = \frac{1}{\sqrt{2}}\left(n_1^\mu + i\lambda_3\, n_2^\mu\right) \ , \tag{111}$$

where the n_i's are given by

$$n_1^\mu = \frac{1}{\sqrt{-2F-M^2}}\left[(p_1^\mu - p_4^\mu) + \frac{(p_1k_2)-(p_1k_3)}{(k_2k_3)}(k_2^\mu - k_3^\mu)\right] \tag{112}$$

$$n_2^\mu = \frac{1}{\sqrt{-2F-M^2}}\frac{\epsilon^{\mu\nu\rho\sigma}k_{2\nu}\,k_{3\rho}\,p_{4\sigma}}{k_2k_3} \ . \tag{113}$$

[10]Note that $-2F-M^2>0$ for physical momenta.

Choosing opposite helicity for the vector boson polarizations one obtains

$$\mathcal{A}^{h_2\to\widetilde{h}_2} = 2\,\mathcal{A}^0\,\left(1+\frac{M^2}{2F}\right)\left(1-\frac{\alpha'(k_2k_3)}{2}\left(2+\frac{M^2}{F}\right)\right) \tag{114}$$

$$\mathcal{A}^{h_2\to\widetilde{h}_3} = 2\,\mathcal{A}^0\,\mathcal{F}_{SV}\,\left(1+\frac{M^2}{2F}\right)\left(\frac{\alpha'(k_2k_3)}{2}\frac{M}{F}\sqrt{-2F-M^2}\right) \tag{115}$$

$$\mathcal{A}^{h_3\to\widetilde{h}_2} = 2\,\mathcal{A}^0\,\left(1+\frac{M^2}{2F}\right)\left(\frac{\alpha'(k_2k_3)}{2}\frac{M}{F}\sqrt{-2F-M^2}\right) \tag{116}$$

$$\mathcal{A}^{h_3\to\widetilde{h}_3} = 2\,\mathcal{A}^0\,\mathcal{F}_{SV}\,\left(1+\frac{M^2}{2F}\right)\left(1+\frac{\alpha'(k_2k_3)}{2}\frac{M^2}{F}\right)\,. \tag{117}$$

with

$$\mathcal{A}^0 = \frac{g^2_{_{YM}}}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)F\;\mathcal{F}_{SV}\,. \tag{118}$$

Note that the diagonal terms, in which there is no polarization flip, expose a $1/k_2k_3$ pole indicating the exchange of a graviton. On the other hand the off diagonal amplitudes do not reveal such a pole. Thus there is no massless particle exchange with the latter choice of polarizations for the massive spin 2 states. In addition the off-diagonal amplitudes contain square roots of kinematical invariants, which seem unusual from a field theoretic point of view. Generically one would expect tree-level amplitudes to exhibit pure pole structure behavior. Note though that these square roots scale with α' and disappear in the field theory limit $\alpha'\to 0$. Thus they might be thought to have stringy origin. Moreover these terms vanish in the formal limit $M\to 0$. It should however be kept in mind that 'kinematical singularities' such as the above square-root cuts are tolerable in field-theory amplitudes with massive particles[11]. In fact a preliminary analysis of the other sector, corresponding to massive spin 2 polarizations h_1, h_4, h_5 mixing with $\tilde{h}_1, \tilde{h}_4, \tilde{h}_5$, displays 'kinematical singularities' to leading order α'^0 with suppression at least $(k_2k_3)/M$.

It should be clear that the limit $M\to 0$ can be only taken in a formal sense, since after all the 1/2 BPS spin 2 states are genuinely massive $M > 2/\sqrt{\alpha'}$. The situation is different for 1/2 BPS spin 1 states which can become massless in the interior of the moduli space. It is relatively easy to investigate the amplitude involving two gauge bosons and two massive 1/2 BPS spin 1 states in connection to possible 'kinematical singularities'. Applying the general formula (85) to $s=1$ one obtains for the scattering of two gauge bosons onto two massive spin 1 states

$$\mathcal{A} = -\frac{g^2_{_{YM}}}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)\frac{1}{k_2k_3}\;\mathcal{F}_{SV}\;\delta^{\perp}_{ij}\,(p_4f_2f_3p_4)\;\Big[-(W_1^i\cdot W_4^j)+(k_2W_1^i)(k_2W_4^j) \tag{119}$$

$$+\,(k_3W_1^i)(k_3W_4^j)-\frac{k_3p_4+1}{k_3p_1}(k_3W_1^i)(k_2W_4^j)-\frac{k_3p_1+1}{k_3p_4}(k_2W_1^i)(k_3W_4^j)\Big]\,. \tag{120}$$

[11]M. B. would like to thank G. Veneziano for an enlightening discussion on this point.

Choosing opposite helicity for the gauge bosons we obtain the following amplitudes

$$\mathcal{A}^{w_1\to\widetilde{w}_1} = -\mathcal{A}^0\ \mathcal{F}_{SV}\ \left(1+\frac{M_W^2}{2F}\right) \tag{121}$$

$$\mathcal{A}^{w_2\to\widetilde{w}_2} = \mathcal{A}^0\ \mathcal{F}_{SV}\ \left(1+\frac{M_W^2}{2F}\right)\left(1+\frac{\alpha'(k_2k_3)}{2}\frac{M_W^2}{F}\right) \tag{122}$$

$$\mathcal{A}^{w_2\to\widetilde{w}_3} = -\mathcal{A}^0\ \mathcal{F}_{SV}\ \left(1+\frac{M_W^2}{2F}\right)\left(\frac{\alpha'(k_2k_3)}{2}\frac{M_W}{F}\sqrt{-2F-M_W^2}\right) \tag{123}$$

$$\mathcal{A}^{w_3\to\widetilde{w}_2} = -\mathcal{A}^0\ \mathcal{F}_{SV}\ \left(1+\frac{M_W^2}{2F}\right)\left(\frac{\alpha'(k_2k_3)}{2}\frac{M_W}{F}\sqrt{-2F-M_W^2}\right) \tag{124}$$

$$\mathcal{A}^{w_3\to\widetilde{w}_3} = \mathcal{A}^0\ \mathcal{F}_{SV}\ \left(1+\frac{M_W^2}{2F}\right)\left(1-\frac{\alpha'(k_2k_3)}{2}\left(2+\frac{M_W^2}{F}\right)\right)\ . \tag{125}$$

All other combinations give vanishing results. As for the massive spin 2 case, we encounter 'kinematical singularities' in the off-diagonal amplitudes that display a universal square-root factor. These amplitudes scale to zero with α' and thus disappear in the field theory limit $\alpha'\to 0$. Moreover, in the limit of vanishing mass $M_W\to 0$ of the 1/2 BPS vector boson these terms vanish. That is exactly what one expects since for scattering of four massless particles the amplitude should not expose this kind of 'kinematical singularities'.

Let us return to the massive spin 2 amplitude, and take the formal limit $\alpha'\to 0$, keeping M fixed. The Shapiro-Virasoro factor $\mathcal{F}_{SV}$ behaves as

$$\mathcal{F}_{SV} = \frac{\Gamma\left[1+\frac{\alpha'}{2}(k_2k_3)\right]\ \Gamma\left[\frac{\alpha'}{2}(k_3p_4)\right]\ \Gamma\left[\frac{\alpha'}{2}(k_3p_1)\right]}{\Gamma\left[2-\frac{\alpha'}{2}(k_2k_3)\right]\ \Gamma\left[-\frac{\alpha'}{2}(k_3p_4)\right]\ \Gamma\left[-\frac{\alpha'}{2}(k_3p_1)\right]} \tag{126}$$

$$\sim \frac{1}{1-\frac{\alpha'}{2}(k_2k_3)}\ \frac{\left[1+\frac{\alpha'}{2}(k_2k_3)\psi(1)\right]\ \left[1+\frac{\alpha'}{2}(k_3p_4)\psi(1)\right]\left[1+\frac{\alpha'}{2}(k_3p_1)\psi(1)\right]}{\left[1-\frac{\alpha'}{2}(k_2k_3)\psi(1)\right]\ \left[1-\frac{\alpha'}{2}(k_3p_4)\psi(1)\right]\left[1-\frac{\alpha'}{2}(k_3p_1)\psi(1)\right]} \tag{127}$$

$$\sim [1+\frac{\alpha'}{2}(k_2k_3)]\,[1+\alpha'(k_2k_3+k_2p_1+k_3p_1)\psi(1)] \sim 1+\frac{\alpha'}{2}(k_2k_3)+\dots\ . \tag{128}$$

due to momentum conservation.

In combination with the other parts of the amplitudes (114) to(117) one obtains for the low energy limit

$$\mathcal{A}^{h_2\to\widetilde{h}_2} = 2\,\frac{g_{_{YM}}^2}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)F\left[\left(1+\frac{M^2}{2F}\right)-\frac{\alpha'}{2}(k_2k_3)\left(1+\frac{M^2}{F}\right)^2+\mathcal{O}\left(\frac{\alpha'}{2}\right)^2\right] \tag{129}$$

$$\mathcal{A}^{h_2\to\widetilde{h}_3} = 2\,\frac{g_{_{YM}}^2}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)\left[\frac{\alpha'}{2}\,(k_2k_3)\left(1+\frac{M^2}{2F}\right)M\sqrt{-2F-M^2}+\mathcal{O}\left(\frac{\alpha'}{2}\right)^2\right] \tag{130}$$

$$\mathcal{A}^{h_3\to\widetilde{h}_2} = 2\,\frac{g_{_{YM}}^2}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)\left[\frac{\alpha'}{2}\,(k_2k_3)\left(1+\frac{M^2}{2F}\right)M\sqrt{-2F-M^2}+\mathcal{O}\left(\frac{\alpha'}{2}\right)^2\right] \tag{131}$$

$$\mathcal{A}^{h_3\to\widetilde{h}_3} = 2\,\frac{g_{_{YM}}^2}{M_s^2}(2\pi)^4\delta(\Sigma_i p_i)F\left[\left(1+\frac{M^2}{2F}\right)+\frac{\alpha'}{2}(k_2k_3)\left(1+\frac{M^2}{F}\right)^2+\mathcal{O}\left(\frac{\alpha'}{2}\right)^2\right] \tag{132}$$

One can also consider the Regge limit of the above amplitudes that corresponds to taking $\alpha'(s-M^2) >> 1$ with t fixed. In this limit the Shapiro Virasoro factor scales like

$$\mathcal{F}_{SV} \approx \frac{\Gamma\left(1-\frac{\alpha'}{4}t\right)}{\Gamma\left(2+\frac{\alpha'}{4}t\right)} e^{+i\pi\alpha' t/4} \left(\alpha' \frac{s-M^2}{4}\right)^{\alpha' t/2} + \ldots \tag{133}$$

One can recognize in order the SV 'form factor', a phase shift and a power law suppression ($t < 0$ in the physical domain). The latter suggest the possibility that higher loop contributions exponentiate in the eikonal approximation. Contrary to what happens for the scattering of gravitons off D-branes [43], the lack of a valid semi-classical limit for 1/2 BPS HS states seems not to favor this interpretation here. Clearly this point deserves further study.

Another interesting limit is the limit in which momentum exchange in the t-channel is extremely small. To analyze this limit it is convenient to work in the Lab frame, where[12] $p_1 = (M, \mathbf{0})$, $k_2 = (E_2, \mathbf{k}_2)$, $-k_3 = (E_3, \mathbf{k}_3)$ and $-p_4 = (E_4, \mathbf{p}_4)$. With that choice the Mandelstam variables assume values

$$s = -(k_2+p_1)^2 = M(M+2E_2) \tag{134}$$

$$t = -(k_2+k_3)^2 = -2E_2E_3(1-\cos\theta) \tag{135}$$

$$u = -(k_3+p_1)^2 = M(M-2E_3)\ . \tag{136}$$

The angular dependance in t can be eliminated by means of the relation $s+t+u = 2M^2$ leading to

$$1-\cos\theta = \frac{M(E_2-E_3)}{E_2E_3} \tag{137}$$

so that (135) becomes $t = -2M(E_2-E_3)$. Let us introduce the variables $Q^2 = -t \geq 0$ and $ME = 2M(E_2+E_3)$. In terms of these new variables and reinstating α' factors the Shapiro-Virasoro factor (88) reads

$$\mathcal{F}_{SV} = \frac{\Gamma(1+\frac{\alpha'}{2}\frac{Q^2}{2})\Gamma(-\frac{\alpha'}{2}\frac{Q^2+EM}{4})\Gamma(-\frac{\alpha'}{2}\frac{Q^2-EM}{4})}{\Gamma(2-\frac{\alpha'}{2}\frac{Q^2}{2})\Gamma(\frac{\alpha'}{2}\frac{Q^2+EM}{4})\Gamma(\frac{\alpha'}{2}\frac{Q^2-EM}{4})} \tag{138}$$

In the limit $Q^2 \to 0$ the Shapiro Virasoro factor scales like

$$\lim_{Q^2\to 0} \mathcal{F}_{SV} \sim 1 + \frac{\alpha'}{2}Q^2\left[\frac{1}{2} - \gamma - \psi\left(\frac{\alpha' EM}{8}\right) - \frac{4}{\alpha' EM} - \frac{\pi}{2}\cot\left(\frac{\pi\alpha' EM}{8}\right) + \mathcal{O}(Q^2)\right]\ , \tag{139}$$

[12] We take all the momenta to be incoming, thus one has to remember that $k_{3,4}^{phys} = -k_{3,4}$. Henceforth we assume $E_{3,4}$ to be the physical energies.

where $\psi(z) = \partial \log \Gamma(z)$ and γ is the Euler-Mascheroni constant. Combining this with the remaining part of the diagonal amplitudes gives

$$\mathcal{A}^{h_2 \to \widetilde{h}_2} = \frac{1}{4}\frac{g_{YM}^2}{M_s^2}(2\pi)^4 \delta(\Sigma_i p_i) \frac{M^2 E^2}{Q^2}\left[1 + Q^2\left(-\frac{4}{E^2}\right)\right. \tag{140}$$

$$\left. + \frac{\alpha'}{2} Q^2 \left(-\frac{1}{2} - \gamma - \psi\left(\frac{\alpha' E M}{8}\right) - \frac{\pi}{2}\cot\left(\frac{\pi \alpha' E M}{8}\right)\right) + \mathcal{O}(Q^2)\right] \tag{141}$$

$$\mathcal{A}^{h_3 \to \widetilde{h}_3} = \frac{1}{4}\frac{g_{YM}^2}{M_s^2}(2\pi)^4 \delta(\Sigma_i p_i) \frac{M^2 E^2}{Q^2}\left[1 + Q^2\left(-\frac{4}{E^2}\right)\right. \tag{142}$$

$$\left. + \frac{\alpha'}{2} Q^2 \left(\frac{1}{2} - \gamma - \psi\left(\frac{\alpha' E M}{8}\right) - \frac{\pi}{2}\cot\left(\frac{\pi \alpha' E M}{8}\right)\right) + \mathcal{O}(Q^2)\right] \ . \tag{143}$$

As expected we find $\alpha' Q^2$ corrections to the massless exchange, due to the infinite tower of string excitations. However even in the low energy limit $\alpha' \to 0$, there is a Q^2 correction to the "minimal" coupling of the higher spin states to the graviton that results from a conspiracy between factors of α' in the numerator and in the denominator and may be interpreted as a remnant of the non-locality of string interactions.

5 Conclusions

Let us conclude by summarizing our results and mentioning open problems. By studying physical processes involving (non)-perturbatively stable 1/2 BPS higher spin states in $\mathcal{N} = 4$ toroidal compactifications of the heterotic string we have gained some insights into their dynamical properties.

Similar results are expected to hold for $\mathcal{N} = 2$ compactifications. In particular our analysis carries over immediately to freely acting orbifolds such as the FHSV model [18, 49] that are expected to receive no quantum corrections to the two-derivative effective action, since $N_h = N_v$ everywhere in the moduli space. Meta-stability of higher spin states is expected in $\mathcal{N} = 1$ compactifications with 'Large Extra Dimensions' [26, 27], that are difficult to accommodate in the heterotic string though [50].

We have also derived explicit expressions for massive non BPS HS states in the first and second massive levels, that couple to pairs of BPS states. Finally, we have analyzed some scattering processes involving BPS HS states, paying attention to spin effects and low-energy limits. Contrary to the recently analyzed case of scattering of gravitons on D-branes [43] for which the eikonal approximation allows to reconstruct the 'classical' geometry, in our case amplitudes are resilient to such a semi-classical approximation, since no supergravity solution can account for 1/2 BPS states with (higher) spin [20, 21, 22, 23, 24]. This seems to raise a puzzle in the identification of the microstates accounting for the entropy of small BH's with two charges [15, 16]. We have pointed out that the

only protected quantity, the helicity super-trace $\mathcal{B}_4 = Str(2h)^4$, makes no distinction between $(2s+1)$ vector multiplets and one spin s supermultiplet. Further analyses of the thermo-dynamical properties of these peculiar HS states is definitely necessary in order to clarify the situation.

The tree-level scattering amplitude of gauge bosons on higher spin states, we studied, exhibits a massless pole and with the appropriate helicity choice for the gauge bosons we extracted the coupling of HS states to the graviton. Equipped with a convenient basis for the polarizations of the HS states, we have investigated interesting limits of the amplitude. For particular choices of the projections the amplitudes reveal unusual 'kinematical singularities', which deserve further study. Moreover, it would be interesting to extend this study to higher orders in perturbation theory and to other HS states, be they BPS and thus stable or not.

Acknowledgments

Discussions with S. Aoyama, M. Bochicchio, G. D'Appollonio, S. Ferrara, D. Forcella, C. Kounnas, J. F. Morales, F. Riccioni, A. Sagnotti, O. Schlotterer, A. Sen, M. Taronna, T. Taylor, G. Veneziano are kindly acknowledged. This work was partially supported by the ERC Advanced Grant n.226455 *"Superfields"* and by the Italian MIUR-PRIN contract 2007-5ATT78 *"Symmetries of the Universe and of the Fundamental Interactions"*.

A Higher Spin 1/2 BPS $\mathcal{N} = 2$ super-multiplets in $D = 4$

As in the $\mathcal{N} = 4$ supersymmetric case discussed in Section 1, in $\mathcal{N} = 2$ supersymmetric compactifications of the Heterotic String, such as the FHSV model [18, 49], the number of complex charged states in 1/2 BPS multiplets with $s \neq 0$ can be easily obtained from the vertex operators

$$V^{(-1)}_{\mathcal{U}_I} = \mathcal{U}_{I,\mu_1\cdots\mu_s} e^{-\varphi}\psi^I e^{i\mathbf{p}_L\mathbf{X}_L} \bar{\partial}X^{\mu_1}_R \cdots \bar{\partial}X^{\mu_s}_R e^{i\mathbf{p}_R\mathbf{X}_R} e^{ipX} \tag{144}$$

with internal excitations $I = 1, 2$ (untwisted directions only!) in the ground state of the L-moving sector, and

$$V^{(-1)}_{\mathcal{U}_\mu} = \mathcal{U}_{\mu,\mu_1\cdots\mu_s} e^{-\varphi}\psi^\mu e^{i\mathbf{p}_L\mathbf{X}_L} \bar{\partial}X^{\mu_1}_R \cdots \bar{\partial}X^{\mu_s}_R e^{i\mathbf{p}_R\mathbf{X}_R} e^{ipX} \tag{145}$$

with space-time excitations in the ground state of the L-moving sector.
The tensors $\mathcal{U}_{I,\mu_1\ldots\mu_s}$ and $\mathcal{U}_{\mu,\mu_1\ldots\mu_s}$ are totally symmetric by construction in the μ_i indices

and, in order for the states to be BRST invariant, they should satisfy

$$p_L^I \mathcal{U}_{I,\mu_1\cdots\mu_s} = p^\mu \mathcal{U}_{I,\mu\mu_2\cdots\mu_s} = \eta^{\mu\nu} \mathcal{U}_{I,\mu\nu\mu_3\cdots\mu_s} = 0 \tag{146}$$

$$p^\mu \mathcal{U}_{\mu,\mu_1\cdots\mu_s} = p^{\mu_1} \mathcal{U}_{\mu,\mu_1\mu_2\cdots\mu_s} = \eta^{\mu_1\mu_2} \mathcal{U}_{\mu,\mu_1\mu_2\mu_3\cdots\mu_s} = 0 \tag{147}$$

The tensor $\mathcal{U}_{I,\mu_1\ldots\mu_s}$ accounts for one[13] states of spin s, while $\mathcal{U}_{\mu,\mu_1\ldots\mu_s}$ gives rise to spin $s+1$, s and $s-1$ states. Therefore the number of bosonic degrees of freedom is

$$n_B = 2(s+1)+1+(1+1)(2s+1)+2(s-1)+1 = (2s+1)4_B \tag{148}$$

Vertex operators for the fermionic states read

$$V^{(-1/2)}_{\Upsilon_{\alpha r}} \quad = \quad \Upsilon_{\alpha r,\mu_1\cdots\mu_s} e^{-\varphi/2} S^\alpha S^r \Sigma_+ e^{i\mathbf{p}_L \mathbf{X}_L} \bar\partial X_R^{\mu_1} \cdots \bar\partial X_R^{\mu_s} e^{i\mathbf{p}_R \mathbf{X}_R} e^{ipX} \tag{149}$$

and

$$V^{(-1/2)}_{\overline{\Upsilon}_{\dot\alpha\dot r}} \quad = \quad \overline{\Upsilon}_{\dot\alpha\dot r,\mu_1\cdots\mu_s} e^{-\varphi/2} C^{\dot\alpha} C^{\dot r} \Sigma_- e^{i\mathbf{p}_L \mathbf{X}_L} \overline{\partial} X_R^{\mu_1} \cdots \overline{\partial} X_R^{\mu_s} e^{i\mathbf{p}_R \mathbf{X}_R} e^{ipX} \tag{150}$$

where S^α, $C^{\dot\alpha}$ are $SO(3,1)$ spin fields and C^r, $C^{\dot r}$ are $SU(2)$ spin fields. BRST invariance requires

$$p^\mu \overline{\sigma}_\mu^{\dot\alpha\alpha} \Upsilon_{\alpha r,\mu_1\cdots\mu_s} + p_{I,L} \tau^I_{rs} \overline{\Upsilon}^{s\dot\alpha}_{\mu_1\cdots\mu_s} = 0 \tag{151}$$

that allows to express $\overline{\Upsilon}^{\dot\alpha\dot r}_{\mu_1\cdots\mu_s}$ in terms of $\Upsilon_{\alpha r,\mu_1\cdots\mu_s}$

$$\overline{\Upsilon}^{A\dot\alpha}_{\mu_1\cdots\mu_s} = \frac{1}{M^2} p^\mu \overline{\sigma}_\mu^{\dot\alpha\alpha} p_L^I \overline{\tau}_I^{rs} \overline{\Upsilon}_{\alpha s,\mu_1\cdots\mu_s} \tag{152}$$

with $M^2 = -p\cdot p = |\mathbf{p_L}|^{\mathbf{2}}$. Combing spin 1/2 from left-movers with spin s from Right-movers one gets $s+1/2$ and $s-1/2$. Taking into account the degeneracy **2** of $SU(2)$, the number of fermionic degrees of freedom turns out to be

$$n_F = 2[2(s+1/2)+1] + 2[2(s-1/2)+1] = (2s+1)4_F \tag{153}$$

Thus, these multiplets contains $(2s+1)(4_B - 4_F)$ complex charged states with maximal spin $s_{hws} = s+1$ and minimal spin $s_{lws} = s-1$.

Similar 1/2 BPS HS $\mathcal{N}=2$ multiplets arise by combining Left-movers 'odd' under twist ('odd' ground states) with Right-movers 'odd' under twist. In $\mathcal{N}=1$ no particle-like 1/2 BPS multiplet is possible. Non BPS $\mathcal{N}=1$ HS multiplets consist in one spin s, two spin $s-1/2$ and one spin $s-1$ [51].

[13]After imposing BRST conditions (146) the internal index I is allowed to run only over the single 'untwisted' direction orthogonal to the central charge p_L^I.

B Details of the amplitude calculation

Here we display all the necessary correlators to compute the 4-point amplitudes in Section 4. Let us start with the left-moving part of the amplitude (78) given by equation (83). Using standard OPE's one gets for the left-moving correlators

$$\langle e^{i\mathbf{p}_L^1 X}(z_1) e^{i\mathbf{p}_L^4 X}(z_4)\rangle = z_{14}^{\mathbf{p}_L^1\,\mathbf{p}_L^4} = z_{14}^{-|\mathbf{p}_L^1|^2} = z_{14}^{-M^2} \tag{154}$$

$$\langle \psi^i(z_1)\,\psi^j(z_4)\rangle = \frac{\delta^{ij}}{z_{14}} \qquad\qquad \langle e^{-\varphi(z_1)} e^{-\varphi(z_4)}\rangle = z_{14}^{-1} \tag{155}$$

and

$$\Big\langle e^{ip_1X}(z_1)\,[\partial X^\kappa - i(k_2\cdot\psi)\psi^\kappa]\,e^{ik_2X}(z_2)\,\big[\partial X^\lambda - i(k_3\cdot\psi)\big]\,\psi^\lambda)e^{ik_3X}(z_3)\,e^{ip_4X}(z_4)\Big\rangle \tag{156}$$

$$= \frac{\prod_{i\neq j} z_{ij}^{k_ik_j}}{z_{23}^2 z_{14}}\,\left[\eta^{\kappa\lambda}(k_2\cdot k_3 - 1) - \frac{z_{14}\,z_{23}}{z_{12}\,z_{34}}k_3^\kappa p_4^\lambda + \frac{z_{14}\,z_{23}}{z_{13}\,z_{24}}p_4^\kappa k_2^\lambda - \frac{z_{14}^2\,z_{23}^2}{z_{12}\,z_{13}\,z_{24}\,z_{34}}p_4^\kappa p_4^\lambda\right]\,. \tag{157}$$

The right-moving part given in equation (84) contains the correlators

$$\Big\langle \overline{J}^{a_2}(\overline{z}_2)\overline{J}^{a_3}(\overline{z}_3)\Big\rangle = \frac{\delta^{a_2a_3}}{\overline{z}_{23}^2} \qquad \Big\langle e^{i\mathbf{p}_R^1X(\overline{z}_1)} e^{i\mathbf{p}_R^4X(\bar{z}_4)}\Big\rangle = \overline{z}_{14}^{\mathbf{p}_R^1\mathbf{p}_R^4} = \overline{z}_{14}^{-|\mathbf{p}_R|^2} = \overline{z}_{14}^{2s-2-M^2} \tag{158}$$

and the correlator

$$\Big\langle \bar\partial X^{\mu_1}\cdots\bar\partial X^{\mu_s} e^{ip_1X}(\bar z_1)\,e^{ik_2X}(\bar z_2)\,e^{ik_3X}(\bar z_3)\,\bar\partial X^{\nu_1}\cdots\bar\partial X^{\nu_s} e^{ip_4X}(\bar z_4)\Big\rangle \tag{159}$$

$$= \frac{\prod_{i\neq j}\bar z_{ij}^{k_ik_j}}{\bar z_{14}^{2s}}\sum_{k=0}^{s}\begin{pmatrix} s\\ k\end{pmatrix}^2 (s-k)!\,(-1)^{s-k}\,\eta^{\mu_{k+1}\nu_{k+1}}...\eta^{\mu_s\nu_s} \tag{160}$$

$$\times\sum_{n=0}^{k}\sum_{m=-n}^{k-2n}\frac{k!\ (K_1^{\mu\nu})^{k-2n-m}\ (K_2^{\mu\nu})^n\ (K_3^{\mu\nu})^{m+n}}{(k-2n-m)!\ n!\ (n+m)!}\left(\frac{\bar z_{12}\bar z_{34}}{\bar z_{13}\bar z_{24}}\right)^m$$

whose derivation is presented in appendix C. Here the K_i's are given by

$$K_1^{\mu\nu} = k_2^\mu\,k_2^\nu + k_3^\mu\,k_3^\nu \qquad K_2^{\mu\nu} = k_2^\mu\,k_3^\nu \qquad K_3^{\mu\nu} = k_3^\mu\,k_2^\nu\ . \tag{161}$$

Combining the left-moving and right-moving part and fixing three vertex operator positions to

$$z_1 = z_\infty = \infty \qquad z_2 = 1 \qquad z_3 = z \qquad z_4 = 0 \tag{162}$$

and after including the c-ghost contribution

$$\Big\langle c(z_1)\,c(z_2)c(z_4)\Big\rangle\,\Big\langle \overline{c}(\overline{z}_1)\,\overline{c}(\overline{z}_2)\overline{c}(\overline{z}_4)\Big\rangle = |z_{12}|^2|z_{14}|^2|z_{24}|^2 \tag{163}$$

we obtain

$$\mathcal{A} = v_{1i} \otimes h_{1\mu_1\dots\mu_s} a_{2\kappa} a_{3\lambda} v_{4j} \otimes h_{4\nu_1\dots\nu_s} \delta^{ij} \int d^2z\, |z|^{2k_3p_4} |1-z|^{2k_2k_3-4} \tag{164}$$

$$\times \left[\eta^{\kappa\lambda}(k_2\cdot k_3 - 1) - \frac{z_{14}\, z_{23}}{z_{12}\, z_{34}} k_3^\kappa p_4^\lambda + \frac{z_{14}\, z_{23}}{z_{13}\, z_{24}} p_4^\kappa k_2^\lambda - \frac{z_{14}^2\, z_{23}^2}{z_{12}\, z_{13}\, z_{24}\, z_{34}} p_4^\kappa p_4^\lambda \right] \mathcal{J}(\bar{z}) \tag{165}$$

with

$$\mathcal{J}(\bar{z}) = \sum_{k=0}^{s} \begin{pmatrix} s \\ k \end{pmatrix}^2 (s-k)!\, (-1)^{s-k}\, \eta^{\mu_{k+1}\nu_{k+1}} \dots \eta^{\mu_s\nu_s} \times \sum_{n=0}^{k} \sum_{m=-n}^{k-2n} \frac{k!\, (K_1^{\mu\nu})^{k-2n-m}\, (K_2^{\mu\nu})^n\, (K_3^{\mu\nu})^{m+n}}{(k-2n-m)!\; n!\; (n+m)!} \bar{z}^m \tag{166}$$

After a few manipulations the amplitude can be written in manifestly gauge invariant form

$$\mathcal{A} = -h_{1\mu_1\dots\mu_s}\, h_{4\nu_1\dots\nu_s} \int d^2z\, |z|^{2k_3p_4} |1-z|^{2k_2k_3-4} \frac{k_2k_3 - 1}{(k_3p_1)(k_3p_4)} v_1\cdot v_4\, (p_4 f_2 f_3 p_4) \mathcal{J}(\bar{z}) \ , \tag{167}$$

where f_i denotes the field strength

$$f_i^{\ \mu\nu} = k_i^\mu a_i^\nu - a_i^\mu k_i^\nu \ . \tag{168}$$

Finally, with the integral

$$\int d^2z |z|^{2\,k_3\,p_4} |1-z|^{2\,k_2\,k_3-4}\, \bar{z}^m = \frac{(k_3\,p_1)(k_3\,p_4)}{(k_2\,k_3-1)(k_2\,k_3)}\, \frac{\Gamma(1+k_2\,k_3)\Gamma(k_3\,p_4)\,\Gamma(k_3\,p_1)}{\Gamma(2-k_2\,k_3)\Gamma(-k_3\,p_4)\,\Gamma(-k_3\,p_1)}\, J_m \tag{169}$$

where J_m is given by[14]

$$J_m = (-1)^m \prod_{r=1}^{|m|} \frac{\theta(m)\,(k_3\,p_4) + \theta(-m)\,(k_3\,p_1) + r}{\theta(m)\,(k_3\,p_1) + \theta(-m)\,(k_3\,p_4) + 1 - r} \tag{170}$$

one obtains

$$\mathcal{A} = -\frac{1}{k_2k_3} v_1\cdot v_4\, (p_4 f_2 f_3 p_4) h_{1\mu_1\dots\mu_s}\, h_{4\nu_1\dots\nu_s}\, \mathcal{F}_{SV}\, \mathcal{J}_R^{\mu_1\dots\mu_s\nu_1\dots\nu_s} \tag{171}$$

with

$$\mathcal{J}_R^{\mu_1\dots\mu_s\nu_1\dots\nu_s} = \sum_{k=0}^{s} \begin{pmatrix} s \\ k \end{pmatrix}^2 (s-k)!\, (-1)^{s-k}\, \eta^{\mu_{k+1}\nu_{k+1}} \dots \eta^{\mu_s\nu_s} \times \sum_{n=0}^{k} \sum_{m=-n}^{k-2n} \frac{k!\, (K_1^{\mu\nu})^{k-2n-m}\, (K_2^{\mu\nu})^n\, (K_3^{\mu\nu})^{m+n}}{(k-2n-m)!\; n!\; (n+m)!} J_m \tag{172}$$

and the Shapiro-Virasoro factor

$$\mathcal{F}_{SV} = \frac{\Gamma(1+k_2\,k_3)\Gamma(k_3\,p_4)\Gamma(k_3\,p_1)}{\Gamma(2-k_2\,k_3)\Gamma(-k_3\,p_4)\Gamma(-k_3\,p_1)} \ . \tag{173}$$

[14]Note that $J_0 = 1$

C HS correlator

Here we present the derivation of the correlator

$$h_{1\mu_1\ldots\mu_s}\, h_{4\nu_1\ldots\nu_s}\, \left\langle \bar\partial X^{\mu_1}\cdots\bar\partial X^{\mu_s} e^{ip_1X}\, e^{ik_2X}\, e^{ik_3X}\, \bar\partial X^{\nu_1}\cdots\bar\partial X^{\nu_s} e^{ip_4X}\right\rangle \tag{174}$$

for an arbitrary s assuming that the polarization rank s tensors h_1 and h_4 are completely symmetric. Rewriting the products of $\overline{\partial}X^{\mu_k}$ by

$$\overline{\partial}X^{\mu_1}...\overline{\partial}X^{\mu_s} = \left[\frac{\overline{\partial}}{\overline{\partial}\alpha_{\mu_1}}...\frac{\overline{\partial}}{\overline{\partial}\alpha_{\mu_s}}\; e^{\alpha\overline{\partial}X}(\overline{z})\right]_{\vec{\alpha}=0} \tag{175}$$

and using the usual OPE's the correlator (174) becomes

$$H_R = \mathcal{I}_R(p_i,\bar z_i)\left[\frac{\partial}{\partial\alpha_{\mu_1}}\cdots\frac{\partial}{\partial\alpha_{\mu_s}}\frac{\partial}{\partial\beta_{\nu_1}}\cdots\frac{\partial}{\partial\beta_{\nu_s}}\; exp\left(-\frac{\alpha\beta}{\bar z_{14}^2} - i\sum_{n\neq1}\frac{\alpha p_n}{\bar z_{1n}} - i\sum_{m\neq4}\frac{\beta p_m}{\bar z_{4m}}\right)\right]_{\substack{\alpha=0\\ \beta=0}} \tag{176}$$

where $\mathcal{I}_R(p_i,\bar z_i)=\prod_{i<j}\bar z_{ij}^{p_ip_j}$ is the Koba-Nielsen factor for the right-moving part. Introducing the variables

$$\widetilde{\alpha}^{\mu} = \alpha^{\mu}+\alpha_0^{\mu}, \qquad\qquad \widetilde{\beta}^{\nu} = \beta^{\nu}+\beta_0^{\nu} \tag{177}$$

with

$$\alpha_0^{\mu} = i\bar z_{14}^2\sum_{m\neq4}\frac{p_m^{\mu}}{\bar z_{4m}} \qquad\qquad \beta_0^{\nu} = i\bar z_{14}^2\sum_{n\neq1}\frac{p_n^{\nu}}{\bar z_{1n}} \tag{178}$$

allows for the compact form

$$\mathcal{I}_R(p_i,\bar z_i)\; exp\left(-\bar z_{14}^2\sum_{\substack{n\neq1\\ m\neq4}}\frac{p_np_m}{\bar z_{1n}\bar z_{4m}}\right)\left[\frac{\partial}{\partial\tilde\alpha_{\mu_1}}\cdots\frac{\partial}{\partial\tilde\alpha_{\mu_s}}\frac{\partial}{\partial\tilde\beta_{\nu_1}}\cdots\frac{\partial}{\partial\tilde\beta_{\nu_s}}\; e^{-\tilde\alpha\tilde\beta/\bar z_{14}^2}\right]_{\substack{\tilde\alpha=\alpha_0\\ \tilde\beta=\beta_0}} \tag{179}$$

Performing the derivatives one gets

$$\begin{aligned}&\mathcal{I}_R(p_i,\bar z_i)\bar z_{14}^{-2s}\sum_{k=0}^{s}\binom{s}{k}^2(s-k)!\,(-1)^{s-k}\,\eta^{\mu_{k+1}\nu_{k+1}}...\eta^{\mu_s\nu_s}\\ &\qquad\times\left(\sum_{n\neq1}\frac{p_n^{\mu_1}}{\bar z_{1n}}\overline{z}_{14}\right)\cdots\left(\sum_{r\neq1}\frac{p_r^{\mu_k}}{\overline{z}_{1r}}\overline{z}_{14}\right)\left(\sum_{m\neq4}\frac{p_m^{\nu_1}}{\overline{z}_{4m}}\overline{z}_{14}\right)\cdots\left(\sum_{q\neq4}\frac{p_q^{\nu_k}}{\overline{z}_{4s}}\overline{z}_{14}\right)\end{aligned} \tag{180}$$

Here we already used the fact that the correlator is symmetric under the exchange of $\mu_i\leftrightarrow\mu_j$ and $\nu_i\leftrightarrow\nu_j$. Now using the BRST constraint $p_i^{\mu_j}h_{\mu_1\ldots\mu_j\ldots\mu_s}=0$ and momentum conservation we can manipulate this to

$$\begin{aligned}\mathcal{W}_R^{s-t}(\bar z_i) &= \frac{\prod_{i\neq j}\bar z_{ij}^{k_ik_j}}{\overline{z}_{14}^{2s}}\sum_{k=0}^{s}\binom{s}{k}^2(s-k)!\,(-1)^{s-k}\,\eta^{\mu_{k+1}\nu_{k+1}}...\eta^{\mu_s\nu_s}\\ &\qquad\times\sum_{n=0}^{k}\sum_{m=-n}^{k-2n}\frac{k!\;(K_1^{\mu\nu})^{k-2n-m}\;(K_2^{\mu\nu})^n\;(K_3^{\mu\nu})^{m+n}}{(k-2n-m)!\;n!\;(n+m)!}\left(\frac{\bar z_{12}\bar z_{34}}{\bar z_{13}\overline{z}_{24}}\right)^m\end{aligned} \tag{181}$$

where

$$K_1^{\mu\nu} = k_2^\mu\, k_2^\nu + k_3^\mu\, k_3^\nu \qquad K_2^{\mu\nu} = k_2^\mu\, k_3^\nu \qquad K_3^{\mu\nu} = k_3^\mu\, k_2^\nu\ . \tag{182}$$

Again we take advantage of the fact that the correlator is symmetric under the exchange of $\mu_i \leftrightarrow \mu_j$ and $\nu_i \leftrightarrow \nu_j$. The product of the respective powers of K_i^n are understood as displayed below

$$(K_1^{\mu\nu})^\alpha\ (K_2^{\mu\nu})^\beta\ (K_3^{\mu\nu})^\gamma = \prod_{u=1}^{\alpha} K_1^{\mu_u \nu_u} \prod_{v=\alpha+1}^{\alpha+\beta} K_2^{\mu_v \nu_v} \prod_{x=\alpha+\beta+1}^{\alpha+\beta+\gamma} K_3^{\mu_x \nu_x}\ . \tag{183}$$

References

[1] D. Chialva, R. Iengo, and J. G. Russo, *Search for the most stable massive state in superstring theory*, JHEP **01** (2005) 001, `arXiv:hep-th/0410152`.

[2] R. Iengo and J. G. Russo, *Handbook on string decay*, JHEP **02** (2006) 041, `arXiv:hep-th/0601072`.

[3] D. Chialva, *Self-interacting fundamental strings and black holes*, Nucl. Phys. **B819** (2009) 256–281, `arXiv:0903.3977 [hep-th]`.

[4] D. Chialva, *String Mass Shifts*, Nucl. Phys. **B819** (2009) 225–255, `arXiv:0903.3979 [hep-th]`.

[5] E. Dudas and J. Mourad, *String theory predictions for future accelerators*, Nucl. Phys. **B575** (2000) 3–34, `arXiv:hep-th/9911019`.

[6] D. Chialva, R. Iengo, and J. G. Russo, *Cross sections for production of closed superstrings at high energy colliders in brane world models*, Phys. Rev. **D71** (2005) 106009, `arXiv:hep-ph/0503125`.

[7] M. Bianchi and A. V. Santini, *String predictions for near future colliders from one-loop scattering amplitudes around D-brane worlds*, JHEP **12** (2006) 010, `arXiv:hep-th/0607224`.

[8] D. Lust, S. Stieberger, and T. R. Taylor, *The LHC String Hunter's Companion*, Nucl. Phys. **B808** (2009) 1–52, `arXiv:0807.3333 [hep-th]`.

[9] L. A. Anchordoqui *et al.*, *LHC Phenomenology for String Hunters*, Nucl. Phys. **B821** (2009) 181–196, `arXiv:0904.3547 [hep-ph]`.

[10] Z. Dong, T. Han, M.-x. Huang, and G. Shiu, *Top Quarks as a Window to String Resonances*, JHEP **09** (2010) 048, `arXiv:1004.5441 [hep-ph]`.

[11] W.-Z. Feng, D. Lust, O. Schlotterer, S. Stieberger, and T. R. Taylor, *Direct Production of Lightest Regge Resonances*, `arXiv:1007.5254 [hep-th]`.

[12] M. Cvetic and D. Youm, *All the Static Spherically Symmetric Black Holes of Heterotic String on a Six Torus*, Nucl. Phys. **B472** (1996) 249–267, `arXiv:hep-th/9512127`.

[13] M. Cvetic and D. Youm, *Entropy of Non-Extreme Charged Rotating Black Holes in String Theory*, Phys. Rev. **D54** (1996) 2612–2620, `arXiv:hep-th/9603147`.

[14] A. W. Peet, *TASI lectures on black holes in string theory*, arXiv:hep-th/0008241.

[15] A. Sen, *Two Charge System Revisited: Small Black Holes or Horizonless Solutions?*, JHEP **05** (2010) 097, arXiv:0908.3402 [hep-th].

[16] A. Dabholkar, J. Gomes, S. Murthy, and A. Sen, *Supersymmetric Index from Black Hole Entropy*, arXiv:1009.3226 [hep-th].

[17] K. S. Narain, M. H. Sarmadi, and E. Witten, *A Note on Toroidal Compactification of Heterotic String Theory*, Nucl. Phys. **B279** (1987) 369.

[18] S. Ferrara, J. A. Harvey, A. Strominger, and C. Vafa, *Second quantized mirror symmetry*, Phys. Lett. **B361** (1995) 59–65, arXiv:hep-th/9505162.

[19] J. G. Russo and L. Susskind, *Asymptotic level density in heterotic string theory and rotating black holes*, Nucl. Phys. **B437** (1995) 611–626, arXiv:hep-th/9405117.

[20] H. Elvang, R. Emparan, D. Mateos, and H. S. Reall, *A supersymmetric black ring*, Phys. Rev. Lett. **93** (2004) 211302, arXiv:hep-th/0407065.

[21] I. Bena and N. P. Warner, *One ring to rule them all ... and in the darkness bind them?*, Adv. Theor. Math. Phys. **9** (2005) 667–701, arXiv:hep-th/0408106.

[22] A. Dabholkar, N. Iizuka, A. Iqubal, A. Sen, and M. Shigemori, *Spinning strings as small black rings*, JHEP **04** (2007) 017, arXiv:hep-th/0611166.

[23] I. Bena and N. P. Warner, *Black holes, black rings and their microstates*, Lect. Notes Phys. **755** (2008) 1–92, arXiv:hep-th/0701216.

[24] N. Iizuka and M. Shigemori, *Are There Four-Dimensional Small Black Rings?*, Phys. Rev. **D77** (2008) 044044, arXiv:0710.4139 [hep-th].

[25] M. Bianchi and L. Lopez, *Pair Production of small Black Holes in Heterotic String Theories*, JHEP **07** (2010) 065, arXiv:1002.3058 [hep-th].

[26] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, Phys. Lett. **B429** (1998) 263–272, arXiv:hep-ph/9803315.

[27] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, Phys. Lett. **B436** (1998) 257–263, arXiv:hep-ph/9804398.

[28] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, *Heterotic String Theory. 1. The Free Heterotic String*, Nucl. Phys. **B256** (1985) 253.

[29] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, *Heterotic String Theory. 2. The Interacting Heterotic String*, Nucl. Phys. **B267** (1986) 75.

[30] M. Bianchi, G. Pradisi, and A. Sagnotti, *Toroidal compactification and symmetry breaking in open string theories*, Nucl. Phys. **B376** (1992) 365–386.

[31] S. Chaudhuri, G. Hockney, and J. D. Lykken, *Maximally supersymmetric string theories in D ¡ 10*, Phys. Rev. Lett. **75** (1995) 2264–2267, `arXiv:hep-th/9505054`.

[32] M. Bianchi, *A note on toroidal compactifications of the type I superstring and other superstring vacuum configurations with 16 supercharges*, Nucl. Phys. **B528** (1998) 73–94, `arXiv:hep-th/9711201`.

[33] E. Witten, *Toroidal compactification without vector structure*, JHEP **02** (1998) 006, `arXiv:hep-th/9712028`.

[34] C. Angelantonj and R. Blumenhagen, *Discrete deformations in type I vacua*, Phys. Lett. **B473** (2000) 86–93, `arXiv:hep-th/9911190`.

[35] C. Bachas, M. Bianchi, R. Blumenhagen, D. Lust, and T. Weigand, *Comments on Orientifolds without Vector Structure*, JHEP **08** (2008) 016, `arXiv:0805.3696 [hep-th]`.

[36] M. Bianchi, J. F. Morales, and H. Samtleben, *On stringy AdS(5) x S**5 and higher spin holography*, JHEP **07** (2003) 062, `arXiv:hep-th/0305052`.

[37] N. Beisert, M. Bianchi, J. F. Morales, and H. Samtleben, *On the spectrum of AdS/CFT beyond supergravity*, JHEP **02** (2004) 001, `arXiv:hep-th/0310292`.

[38] N. Beisert, M. Bianchi, J. F. Morales, and H. Samtleben, *Higher spin symmetry and N = 4 SYM*, JHEP **07** (2004) 058, `arXiv:hep-th/0405057`.

[39] A. Hanany, D. Forcella, and J. Troost, *The covariant perturbative string spectrum*, `arXiv:1007.2622 [hep-th]`.

[40] Y. Tanii and Y. Watabiki, *VERTEX FUNCTIONS IN THE PATH INTEGRAL FORMALISM OF OPEN STRING THEORIES*, Int. J. Mod. Phys. **A3** (1988) 2601–2620.

[41] I. G. Koh, W. Troost, and A. Van Proeyen, *COVARIANT HIGHER SPIN VERTEX OPERATORS IN THE RAMOND SECTOR*, Nucl. Phys. **B292** (1987) 201.

[42] E. Kiritsis, *String theory in a nutshell*,. Princeton, USA: Univ. Pr. (2007) 588 p.

[43] G. D'Appollonio, P. Di Vecchia, R. Russo, and G. Veneziano, *High-energy string-brane scattering: leading eikonal and beyond*, `arXiv:1008.4773 [hep-th]`.

[44] A. Sagnotti and M. Taronna, *String Lessons for Higher-Spin Interactions*, `arXiv:1006.5242 [hep-th]`.

[45] A. Fotopoulos and M. Tsulaia, *On the Tensionless Limit of String theory, Off - Shell Higher Spin Interaction Vertices and BCFW Recursion Relations*, `arXiv:1009.0727 [hep-th]`.

[46] A. Fotopoulos and N. Prezas, *Pomerons and BCFW recursion relations for strings on D- branes*, `arXiv:1009.3903 [hep-th]`.

[47] M. Bianchi, H. Elvang, and D. Z. Freedman, *Generating Tree Amplitudes in N=4 SYM and N = 8 SG*, JHEP **09** (2008) 063, `arXiv:0805.0757 [hep-th]`.

[48] S. Y. Choi, J. S. Shim, and H. S. Song, *Factorization and polarization in linearized gravity*, Phys. Rev. **D51** (1995) 2751–2769, `arXiv:hep-th/9411092`.

[49] A. Klemm and M. Marino, *Counting BPS states on the Enriques Calabi-Yau*, Commun. Math. Phys. **280** (2008) 27–76, `arXiv:hep-th/0512227`.

[50] K. Benakli and Y. Oz, *Small instantons and weak scale string theory*, Phys. Lett. **B472** (2000) 83–88, `arXiv:hep-th/9910090`.

[51] Y. M. Zinoviev, *Massive supermultiplets with spin 3/2*, JHEP **05** (2007) 092, `arXiv:hep-th/0703118`.